

Cloud Physics. Exercises

Synoptic/mesoscale meteorology

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Finite difference (FD) approximations

» $y = y(t)$

» FD approximation: $\frac{dy}{dt} \approx \frac{\Delta y}{\Delta t}$

» FD approximations are useful for solving differential equations, E.g. suppose y obeys:

» $\frac{dy}{dt} = g(y, t)$

» $\frac{dy}{dt} = g(y, t) \approx \frac{\Delta y}{\Delta t} \implies \Delta y = \Delta t \times g(y, t)$

- If we know the value of y at $t = 0$, then by marching through time in time-steps of constant interval, Δt , we can update y at each time, $t > 0$



EXERCISE 1: SIMULATE GROWTH OF A RAINDROP



Basic equations

Growth of raindrop (r , m_r)

$$\frac{Dm_r}{Dt} \approx E \pi r^2 v_t \text{LWC}$$

*Mass concentration
(kg/m^3) of cloud-liquid*

Fall-speed of raindrop (cm/sec)

$$v_t = k \sqrt{r}$$

At MSL, $k = 2200 \text{ cm}^{1/2} \text{ s}^{-1}$

mass of raindrop (kg)

$$m_r = \rho_L \frac{4}{3} \pi r^3$$

Density of liquid = 1000 kg m^{-3}



Idealised case to simulate

- » Uniform, deep updraft
 - $LWC = 3 \text{ g m}^{-3}$ and $w = 5 \text{ m s}^{-1}$ at all levels in updraft
- » Drop is initially 0.5 mm in radius at 2 km above cloud-base
- » Neglect condensation and evaporation, assume collision efficiency of unity
- » Write FD scheme in MATLAB for drop growth
- » Plot size and height as a function of time
- » How long does it take for the drop to fall out of the cloud, and what is its final size ?



Technique: (1) use finite-difference (FD) approximation to replace derivative

$$(m_{i+1} - m_i)/(t_{i+1} - t_i) = \dots$$

(2) re-arrange FD equation with terms for next time on LHS and current times on RHS

$$m_{i+1} = m_i + \Delta t \dots$$

(3) march through times



- » Was it justifiable to neglect condensation for growth of drop ?
- » Assuming constant supersaturation of 1% (saturation ratio, $S = 1.01$), repeat above simulation by including condensation

- » If only condensation, then

$$\frac{rdr}{dt} = (S - 1) \times \zeta$$

- » $\zeta = 70 \mu\text{m}^2 \text{sec}^{-1}$



Fall-out of drop through cloud-free environment

- » Environment below cloud-base has constant relative humidity (e.g. 80%) and cloud-base is 2 km above ground.
- » Predict whether drop would survive evaporation and reach the ground, if falling out from cloud at various sizes. Include only evaporation, neglect coalescence in cloud-free air.
 - Plot distance fallen until complete evaporation, as a function of initial size, for various drops.



EXERCISE 2: CREATE PARCEL MODEL



Assumptions

- » 1D upward motion of a parcel can provide plausible vertical profiles
- » Adiabatic
- » Pressures inside and outside are always equal
- » Constant rate of ascent
- » Simplified microphysics, depending on exercises



Create a parcel model of cloud

- » Open **MATLAB**
- » Solve the equation for the ascent of a parcel rising at a fixed rate:
 - **$Dz/Dt = w = \text{constant}$**
- » Technique: (1) use finite-difference (FD) approximation to replace derivative
 - » $(z_{i+1} - z_i)/(t_{i+1} - t_i) = w$
 - (2) re-arrange FD equation with terms for next time on LHS and current times on RHS
 - » $z_{i+1} = z_i + w \Delta t$
 - (3) march through times



MATLAB code for parcel model

- » $dt = 5$
- » $w = 5$
- » $z(1) = 100$
- » For $i=1:200$
- » $z(i+1) = z(i) + w * dt$
- » $t(i+1) = t(i) + dt$
- » End
- » Plot(t,z)



Include temperature, T , in parcel if always dry or if always saturated

» **$-dT/dz = \Gamma \sim \text{constant}$**

» Lapse rate, Γ , ~ 6 K / km if saturated (either constant or exact formula from notes) and 9.8 K/km if dry

» FD equation:

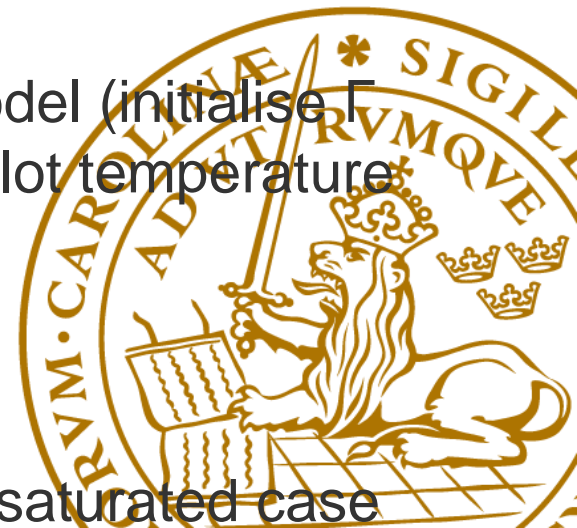
$$- (T_{i+1} - T_i) / (z_{i+1} - z_i) = \Gamma$$

$$- T_{i+1} = T_i - \Gamma \Delta z$$

» Add 4 lines of **MATLAB** code into parcel model (initialise Γ and T_1 , do FD equation at each level, and plot temperature profile)

– Use standard atmosphere table

» Run model once for dry case and again for saturated case



Include pressure in parcel

» $p = p_0 \exp(-z / H)$

» $H = 7300$ metres



Include water vapour in parcel, if always saturated or always unsaturated

» $D Q_v / Dt = S$

» Vapour mixing ratio:

– $Q_v = Q_T = \text{constant when dry}$

$Q_v = Q_{v,s}(p, T)$ when saturated

– $Q_{v,s}(p, T) = \varepsilon e_s(T) / T$

» Hints for MATLAB code:

– $qvs(i) = A * \exp(-B/temp(i)) * \text{eps} / \text{pres}(i)$

– $Qv(i) = qvs(i)$

– Or if unsaturated: $Qv(i) = Qv(1)$



Include cloud-liquid mass in parcel

» $D Q_T / Dt = 0$

» $Q_T = Q_v + Q_w$

» Hints for Matlab code:

– $qt(i) = qt(1)$

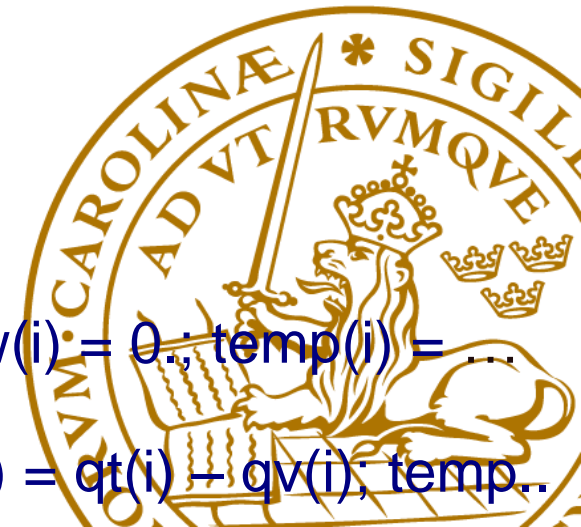
– $lf(qv(i) \geq qvs(i))$

 » $sat_flag = 1;$

– end

– If unsaturated ($sat_flag=0$): $qv(i) = qt(i); qw(i) = 0.; temp(i) = \dots$

– If saturated ($sat_flag=1$): $qv(i) = qvs(i); qw(i) = qt(i) - qv(i); temp..$



Plots to create and print

- » Perform a simulation of a parcel rising from 100 metres altitude, where it has properties of standard atmosphere initially.
 - Initially clear, becoming cloudy during ascent
- » Plot vertical profiles of temperature, pressure, height, vapour mixing ratio, cloud-liquid mixing ratio, cloud-liquid content, saturated mixing ratio, and relative humidity for 5 km of ascent.
- » Repeat plots for various ascent rates: 1, 10, 30 m/s
 - What is the effect on adiabatic liquid water content ?



Include droplet number mixing ratio, n_w , in parcel, due to aerosol activation

- » Assume all droplets are initiated at cloud-base, neglect rain
- » Activity spectrum for a given aerosol conditions (C = normalised aerosol concentration)
 - N_w = number concentration of droplets when percentage supersaturation, s , is increased from $s = 0$
 - **$s = 100 (e / e_s(T) - 1)$**
 - **$N_w = C s^k$**
 - *Twomey formula for supersaturation peak at cloud-base*
 - **$D n_w / Dt = 0$**



Include droplet number mixing ratio, n_w , in parcel, due to aerosol activation (cont.)

- » Use typical value of C for 'green-ocean' or polluted conditions of aerosol in Amazon rainforest
- » Hints for **MATLAB** code
 - Unsaturated: $n_w(i) = 0$
 - Saturated: $n_w(i+1) = n_w(i)$ if $n_w(i)$ is positive;
 - » or just $n_w(i) = C * \text{power}(\text{supersat}, k)$ if $n_w(i)$ is zero.



Plots to create and print

- » Repeat standard simulation of parcel rising from 100 metres altitude at 5 m/s
- » Plot vertical profiles of droplet number concentration (per cm³) and of average droplet size, assuming all droplets are same size
- » Now produce the same plots with other ascent speeds: 1, 10 , 30 m/sec and for other aerosol conditions (x 0.1 and x 10).
 - How does droplet size depend on ascent and aerosol loading ?



Include rain formation

» $DQ_r / Dt = S + A - F$

» Autoconversion from cloud-liquid to rain: $S = C_1 (Q_w - Q_{w,crit})$ if mean droplet size > 20 microns

– $Q_{w,crit}$ = cloud-liquid m. r. for a 20-micron droplet

» Accretion of cloud-liquid: $A = C_2 Q_r Q_w$

» Fall-out : $F = Q_r / \tau$

» Use suggested values of the constants, or derive your own estimates from equations in lectures



Include rain formation (cont.)

- » Add to your earlier plots, the vertical profile of rain mixing ratio, for various aerosol conditions and ascent rates
- » Discuss how your model shows an impact on rain formation by higher aerosol pollution
 - How does the level of rain formation change between
 - » green-ocean and polluted aerosol conditions for Brazilian rainforest ?
 - » and for Sao Paulo urban area ?



Feeling bored ? Just ask for extra work

» Include in parcel model:

- entrainment of environmental air (standard atmosphere) in parcel model
 - » Include entrainment term on RHS of all evolution equations
- ascent determined by buoyancy,
 - » ascent forced to be never less than a threshold



Feeling bored ? Just ask for extra work

- in-cloud supersaturation as a function of ascent
- in-cloud droplet activation
- Better microphysics for rain (Khairoutdinov and Kogan 2000)

