Cloud Physics. Exercises

Synoptic/mesoscale meteorology

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Finite difference (FD) approximations

» $y = y(t)$

» FD approximation: $\dfrac{dy}{dt} \approx \dfrac{\Delta y}{\Delta t}$

» FD approximations are useful for solving differential equations, E.g. suppose *y* obeys:

- » $\dfrac{dy}{dt} = g(y,t)$
- » $\dfrac{dy}{dt} = g(y,t) \approx \dfrac{\Delta y}{\Delta t} \Longrightarrow \Delta y = \Delta \mathrm{t} \times \mathrm{g}(\mathrm{y},\mathrm{t})$
  - – If we know the value of *y* at *t = 0*, then by marching through time in time-steps of constant interval, Δt, we can update y at each time, *t* > 0

EXERCISE 1: SIMULATE GROWTH OF A RAINDROP

Basic equations

Growth of raindrop (r, m_r)

$$\frac{Dm_r}{Dt} \approx E\,\pi\,r^2\,v_t\,LWC$$

Mass concentration (kg/m^3) of cloud-liquid

Fall-speed of raindrop (cm/sec)

$$v_t = k\sqrt{r}$$

At MSL, k =2200 cm ^{½} s^{-1}

mass of raindrop (kg)

$$m_r = \rho_L \frac{4}{3} \pi\, r^3$$

Density of liquid = 1000 kg m $^{-3}$

Idealised case to simulate

» Uniform, deep updraft

  – LWC = 3 g m^{-3} and w = 5 m s^{-1} at all levels in updraft

» Drop is initially 0.5 mm in radius at 2 km above cloud-base

» Neglect condensation and evaporation, assume collision efficiency of unity

» Write FD scheme in MATLAB for drop growth

» Plot size and height as a function of time

» How long does it take for the drop to fall out of the cloud, and what is its final size ?

Technique: (1) use finite-difference (FD) approximation to replace derivative

$$(m_{i+1} - m_i)/(t_{i+1} - t_i) = \ldots$$

(2) re-arrange FD equation with terms for next time on LHS and current times on RHS

$$m_{i+1} = m_i + \Delta t \ldots$$

(3) march through times

- Was it justifiable to neglect condensation for growth of drop ?
- Assuming constant supersaturation of 1% (saturation ratio, S = 1.01), repeat above simulation by including condensation
  - If only condensation, then

$$\frac{rdr}{dt} = (S - 1) \times \zeta$$

  - $\zeta = 70$ µm^2 sec^{-1}

Fall-out of drop through cloud-free environment

» Environment below cloud-base has constant relative humidity (e.g. 80%) and cloud-base is 2 km above ground.

» Predict whether drop would survive evaporation and reach the ground, if falling out from cloud at various sizes. Include only evaporation, neglect coalescence in cloud-free air.

  – Plot distance fallen until complete evaporation, as a function of initial size, for various drops.

EXERCISE 2: CREATE PARCEL MODEL

Assumptions

- 1D upward motion of a parcel can provide plausible vertical profiles
- Adiabatic
- Pressures inside and outside are always equal
- Constant rate of ascent
- Simplified microphysics, depending on exercises

Create a parcel model of cloud

» Open MATLAB

» Solve the equation for the ascent of a parcel rising at a fixed rate:

– **Dz/Dt = w = constant**

» Technique: (1) use finite-difference (FD) approximation to replace derivative

» $(z_{i+1} - z_i)/(t_{i+1} - t_i) = w$

– (2) re-arrange FD equation with terms for next time on LHS and current times on RHS

» $z_{i+1} = z_i + w\,\Delta t$

– (3) march through times

MATLAB code for parcel model

» dt = 5

» w = 5

» z(1) = 100

» For i=1:200

» z(i+1) = z(i) + w * dt

» t(i+1) = t(i) + dt

» End

» Plot(t,z)

Include temperature, *T*, in parcel if always dry or if always saturated

- **-dT/dz = Γ ~ constant**

- Lapse rate, **Γ,** ~ 6 K / km if saturated (either constant or exact formula from notes) and 9.8 K/km if dry

- FD equation:
  - $-(T_{i+1} - T_i)/(z_{i+1} - z_i) = \Gamma$
  - $T_{i+1} = T_i - \Gamma \Delta z$

- Add 4 lines of MATLAB code into parcel model (initialise Γ and T_1, do FD equation at each level, and plot temperature profile)
  - Use standard atmosphere table

- Run model once for dry case and again for saturated case

Include pressure in parcel

» $p = p_0 \exp(- z / H)$

» $H = 7300$ metres

Include water vapour in parcel, if always saturated or always unsaturated

» **$D\,Q_v \,/\, Dt = S$**

» Vapour mixing ratio:

– $Q_v = Q_T$ = constant when dry

$Q_v = Q_{v,s}(p, T)$ when saturated

– $Q_{v,s}(p, T) = \varepsilon\, e_s(T) \,/\, T$

» Hints for MATLAB code:

– qvs(i) = A * exp(-B/temp(i)) * eps/ pres(i)

– Qv(i) = qvs(i)

– Or if unsaturated: Qv(i) = Qv(1)

Include cloud-liquid mass in parcel

- **$D\,Q_T / Dt = 0$**
- **$Q_T = Q_v + Q_w$**

- Hints for Matlab code:
  - qt(i) = qt(1)
  - If(qv(i) >= qvs(i))
    - sat_flag = 1;
  - end
  - If unsaturated (sat_flag=0): qv(i) = qt(i); qw(i) = 0.; temp(i) = …
  - If saturated (sat_flag=1): qv(i) = qvs(i); qw(i) = qt(i) – qv(i); temp..

Plots to create and print

- Perform a simulation of a parcel rising from 100 metres altitude, where it has properties of standard atmosphere initially.
  - Initially clear, becoming cloudy during ascent
- Plot vertical profiles of temperature, pressure, height, vapour mixing ratio, cloud-liquid mixing ratio, cloud-liquid content, saturated mixing ratio, and relative humidity for 5 km of ascent.
- Repeat plots for various ascent rates: 1, 10, 30 m/s
  - What is the effect on adiabatic liquid water content ?

Include droplet number mixing ratio, n_w, in parcel, due to aerosol activation

» Assume all droplets are initiated at cloud-base, neglect rain

» Activity spectrum for a given aerosol conditions (C = normalised aerosol concentration)

– N_w = number concentration of droplets when percentage supersaturation, *s*, is increased from *s = 0*

– $s = 100\,(e / e_s(T) - 1)$

– $N_w = C\,s^{k}$

– *Twomey formula for supersaturation peak at cloud-base*

– $D\,n_w / Dt = 0$

Include droplet number mixing ratio, n_w, in parcel, due to aerosol activation (cont.)

» Use typical value of *C* for ‘green-ocean’ or polluted conditions of aerosol in Amazon rainforest

» Hints for MATLAB code

– Unsaturated: nw(i) = 0

– Saturated: nw(i+1) = nw(i) if nw(i) is positive;

» or just nw(i) = C * power(supersat, k) if nw(i) is zero.

Plots to create and print

- Repeat standard simulation of parcel rising from 100 metres altitude at 5 m/s

- Plot vertical profiles of droplet number concentration (per cm3) and of average droplet size, assuming all droplets are same size

- Now produce the same plots with other ascent speeds: 1, 10 , 30 m/sec and for other aerosol conditions (x 0.1 and x 10).

  - How does droplet size depend on ascent and aerosol loading ?

Include rain formation

» $DQ_r / Dt = S + A - F$

» Autoconversion from cloud-liquid to rain: $S = C_1 (Q_w - Q_{w,crit})]$ if mean droplet size > 20 microns

– $Q_{w,crit}$ = cloud-liquid m. r. for a 20-micron droplet

» Accretion of cloud-liquid: $A = C_2 Q_r Q_w$

» Fall-out : $F = Q_r / \tau$

» Use suggested values of the constants, or derive your own estimates from equations in lectures

Include rain formation (cont.)

» Add to your earlier plots, the vertical profile of rain mixing ratio, for various aerosol conditions and ascent rates

» Discuss how your model shows an impact on rain formation by higher aerosol pollution

– How does the level of rain formation change between

» green-ocean and polluted aerosol conditions for Brazilian rainforest ?

» and for Sao Paulo urban area ?

Feeling bored ? Just ask for extra work

- Include in parcel model:
  - entrainment of environmental air (standard atmosphere) in parcel model
    - Include entrainment term on RHS of all evolution equations
  - ascent determined by buoyancy,
    - ascent forced to be never less than a threshold

Feeling bored ? Just ask for extra work

- in-cloud supersaturation as a function of ascent
- in-cloud droplet activation
- Better microphysics for rain (Khairoutdinov and Kogan 2000)