Introduction to Cloud Modeling. Part I: Model Architecture and Parameterizations

Vaughan Phillips

Course on “atmospheric aerosols and clouds with introduction to process oriented modeling”,
Sao Paulo University

Outline

» Overview

» Initial value problem

» Finite-difference equations and discretization

» Parameterization

» Summary

OVERVIEW

Why use numerical models to study clouds ?

» Huge range of spatial scales

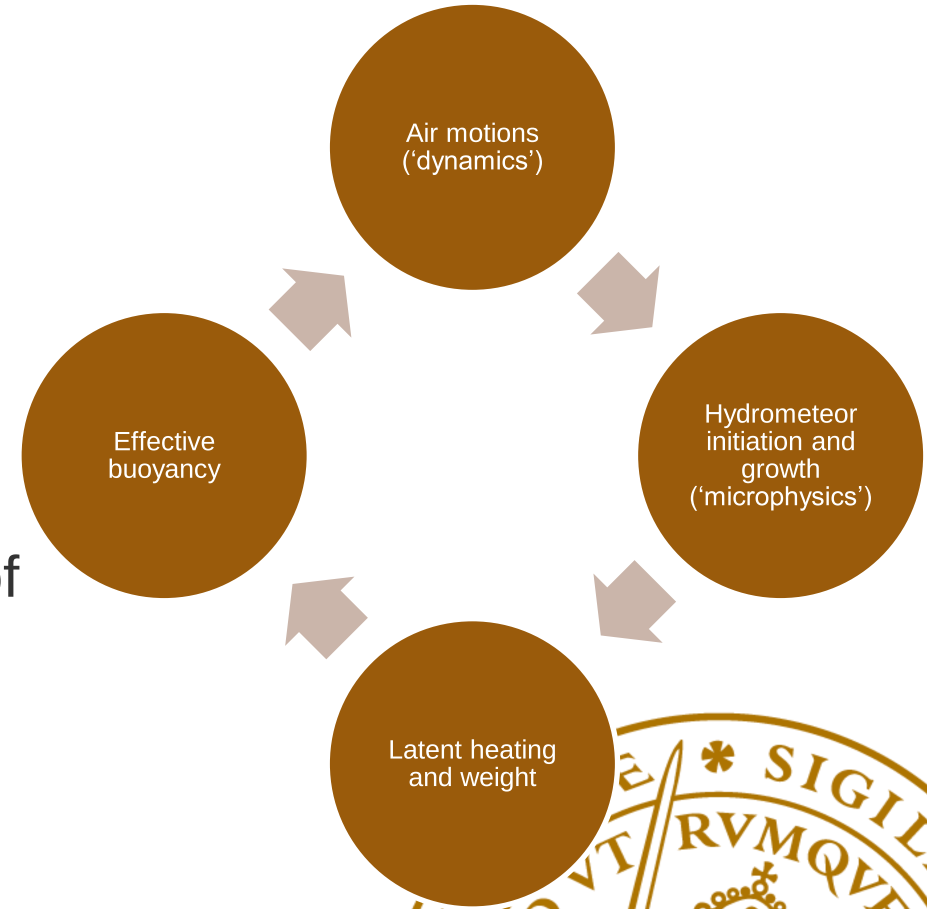
– lab studies of cloud difficult

» Complex feedbacks between microphysics and dynamics in clouds

– Latent heating and burden of condensate affects vertical motions (F_B)

– Vertical motions determine precipitation production and microphysical processes

» Models allow each feedback mechanism to be studied

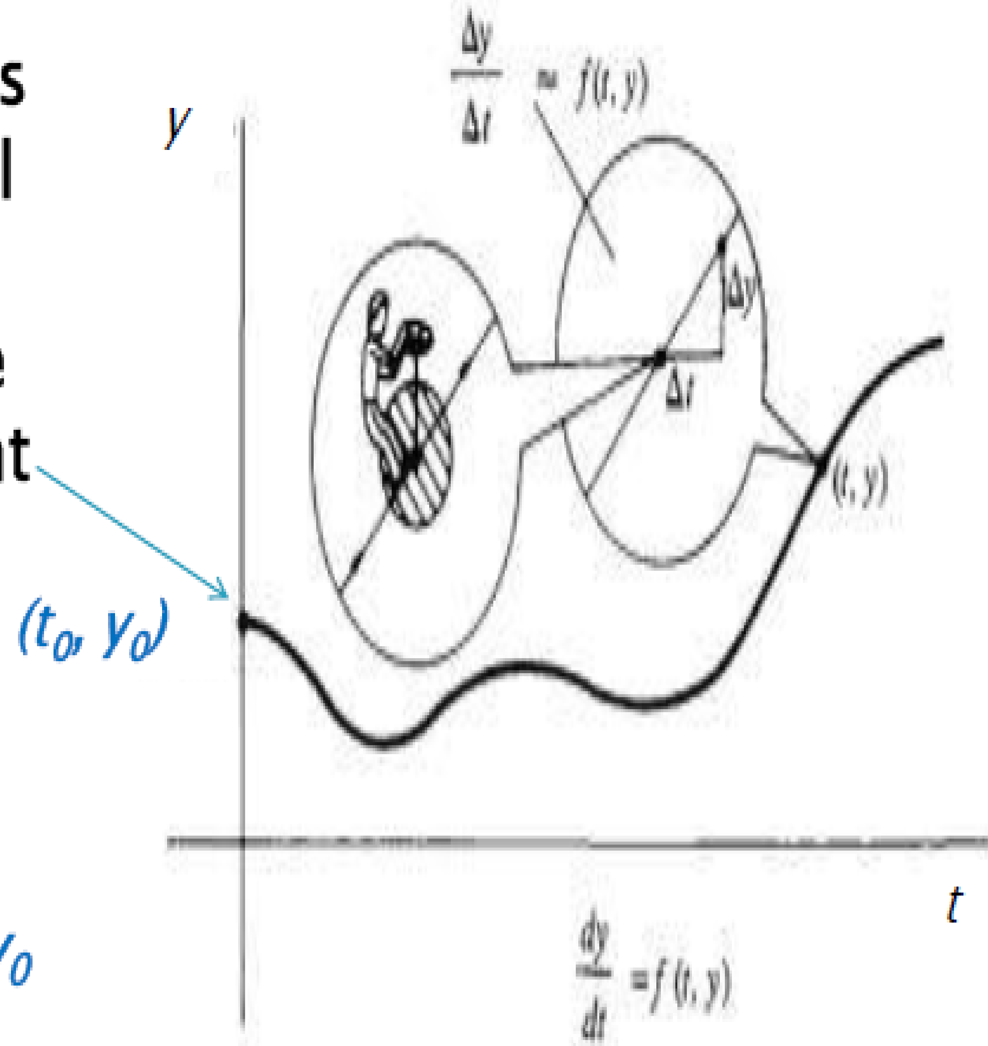


INITIAL VALUE PROBLEM

maths definition: Initial Value Problem

- **Initial value problem** is an ordinary differential equation (ODE) with a known, specified value of solution at one point (the initial condition)

- Simple example:
  - ODE: $y'(t) = f(t, y(t))$
  - initial condition: $y(t_0) = y_0$

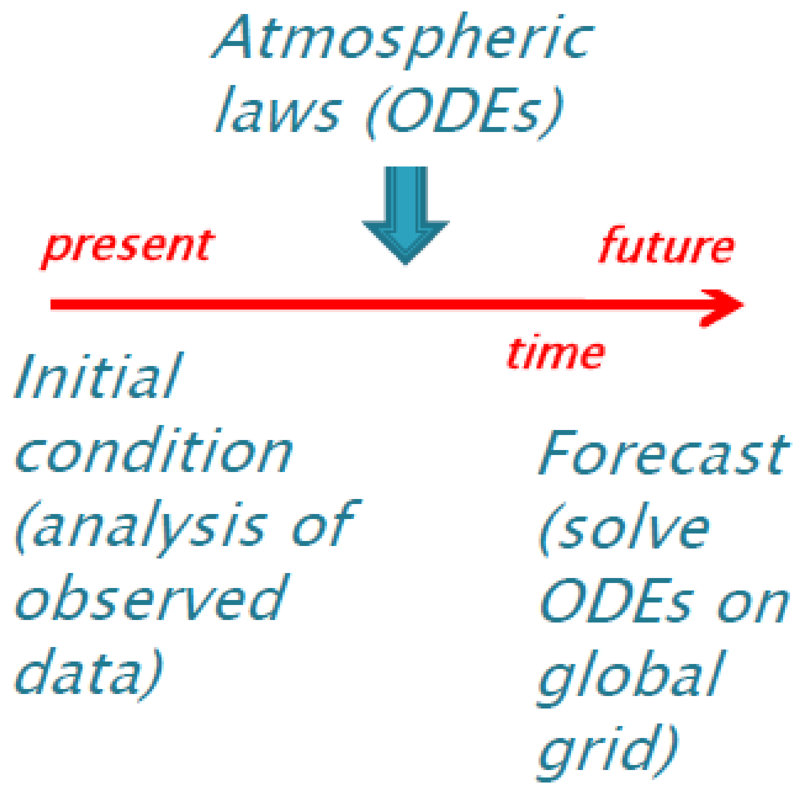


Atmospheric simulation is an initial value problem with laws being ODEs

- **Momentum:** $Dv/Dt = \Sigma_i F_i$
  - v is velocity and F_i are forces per unit mass

- **Heat:** $D\theta / Dt = \Sigma_i S_i$
  - θ is potential temperature;
  - S_i are heat-sources (e.g. radiative);
  - 1st Law of thermodynamics

- **Mass:** $D\rho/Dt = -\rho \ div$
  - ρ is air density
  - div (= $\nabla.v$) is divergence of v



FINITE-DIFFERENCE EQUATIONS AND DISCRETIZATION

maths facts: Nonlinear Equations

- **Any equation is linear if it can be expressed as $f(x) = C$ where**

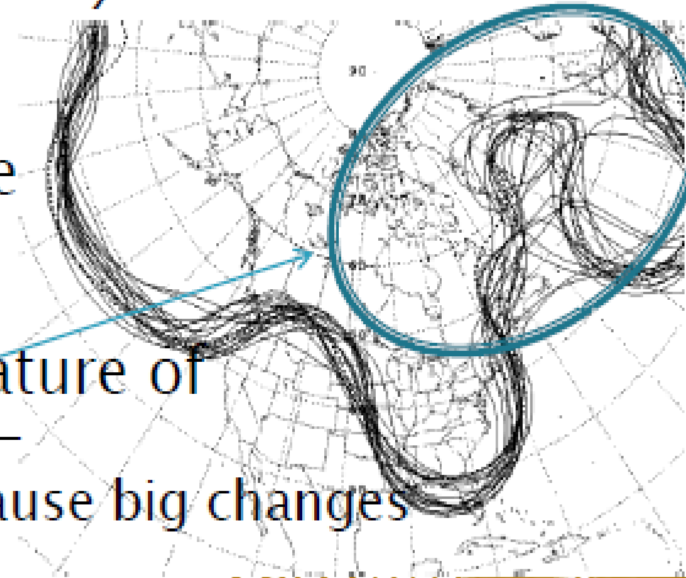
  $$f(a_1x_1 + a_2x_2) = a_1f(x_1) + a_2f(x_2)$$

  for all x_1 and x_2 (here, a_1, a_2 and C are constants); **the equation is nonlinear otherwise**

  - Eg. $3x = 2$ is linear, but $x^2 + 3x = 2$ is not
  - Eg. $dy/dt = -y$ is linear but $dy/dt = -y^2$ is nonlinear

- Nonlinear ODEs are difficult to solve
  - **No analytical solution,** usually

- **Sensitivity to initial conditions** is feature of **chaotic** systems of nonlinear ODEs:-
  - Small changes to one part of system cause big changes throughout it



maths method: Numerical Solution of ODEs

- Approximate the nonlinear ODEs:
  - **Discretisation**: solution estimated only at discrete points/times on grid
  - **Finite differences**: derivatives in ODEs replaced by approximations (e.g. by Taylor expansions)
- Computers for numerical solution of difference equation

maths method: Discretization and finite differences to solve ODE numerically by computer

Example of initial value problem : Initial condition, $y(t_0, x) = Y_0(x)$
Solution: $y = y(t, x)$ ODE: $y'(t) = f(t, x)$

- Discretisation: $y_n^j = y(x^j, t_n)$ where $x^j = j\,\Delta x$ and $t_n = n\,\Delta t$

- **Derivatives approximated** by finite differences; e.g.:

$$y'^j_n \approx (y_{n+1}^j - y_n^j) / \Delta t$$

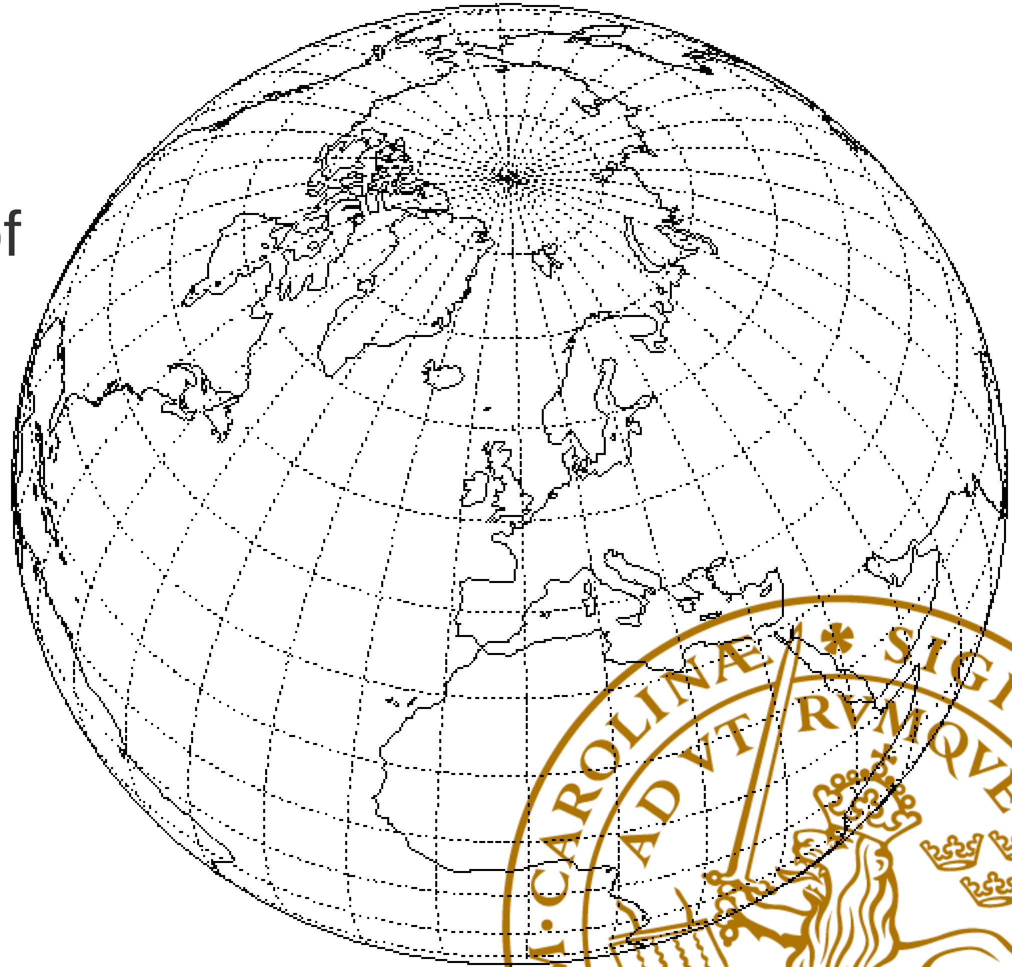
- **Difference equation** replaces ODE:

$$y_{n+1}^j = \Delta t\, f(t_n, x^j) + y_n^j$$

- At $n = 0$, use $y_0^j = Y_0(x^j)$; then **increment *n* successively,** updating y_{n+1}^j by marching through time in time-steps, Δt

Atmospheric Models

- » Global models divide the world into a grid, data is held on the intersections of the grid.
- » Figure shows a 10° grid
- » MetOffice global model uses a grid of approximately 0.8° longitude by 0.5° latitude
  - – UK represented by ~10x20 grid points (~60km spacing)



Mesoscale (e.g. cloud) models now used for weather forecasting

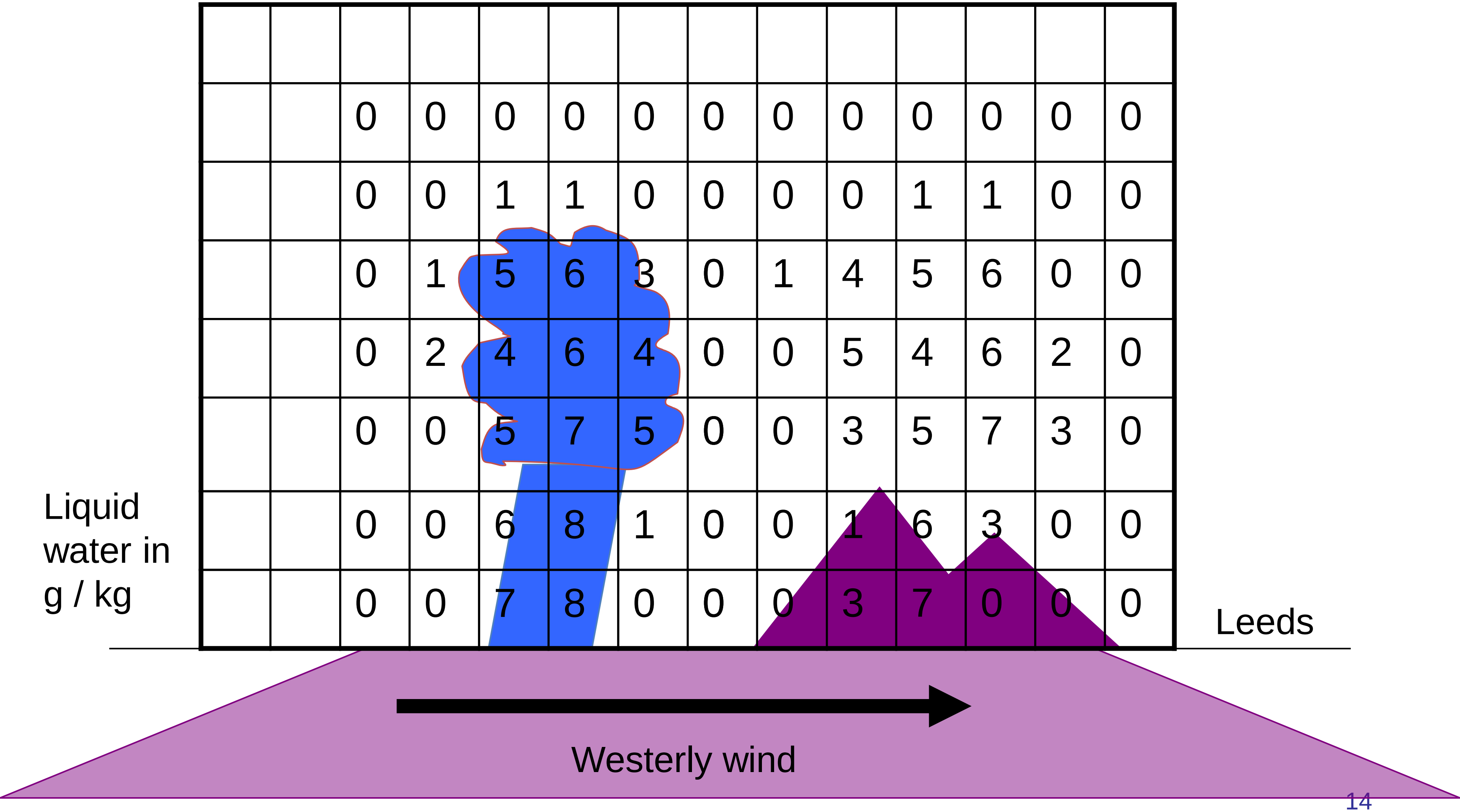
» MetOffice mesoscale model – higher resolution, limited area model.

» Uses the global model to provide initialization and boundary conditions.

  – 0.11° by 0.11° grid: approximately 11km resolution.

  – 38 vertical levels in both global and mesoscale models, spacing increases with altitude.

		0	0	0	0	0	0	0	0	0	0	0	0
		0	0	1	1	0	0	0	0	1	1	0	0
		0	1	5	6	3	0	1	4	5	6	0	0
		0	2	4	6	4	0	0	5	4	6	2	0
		0	0	5	7	5	0	0	3	5	7	3	0
		0	0	6	8	1	0	0	1	6	3	0	0
		0	0	7	8	0	0	0	3	7	0	0	0



PARAMETERIZATION

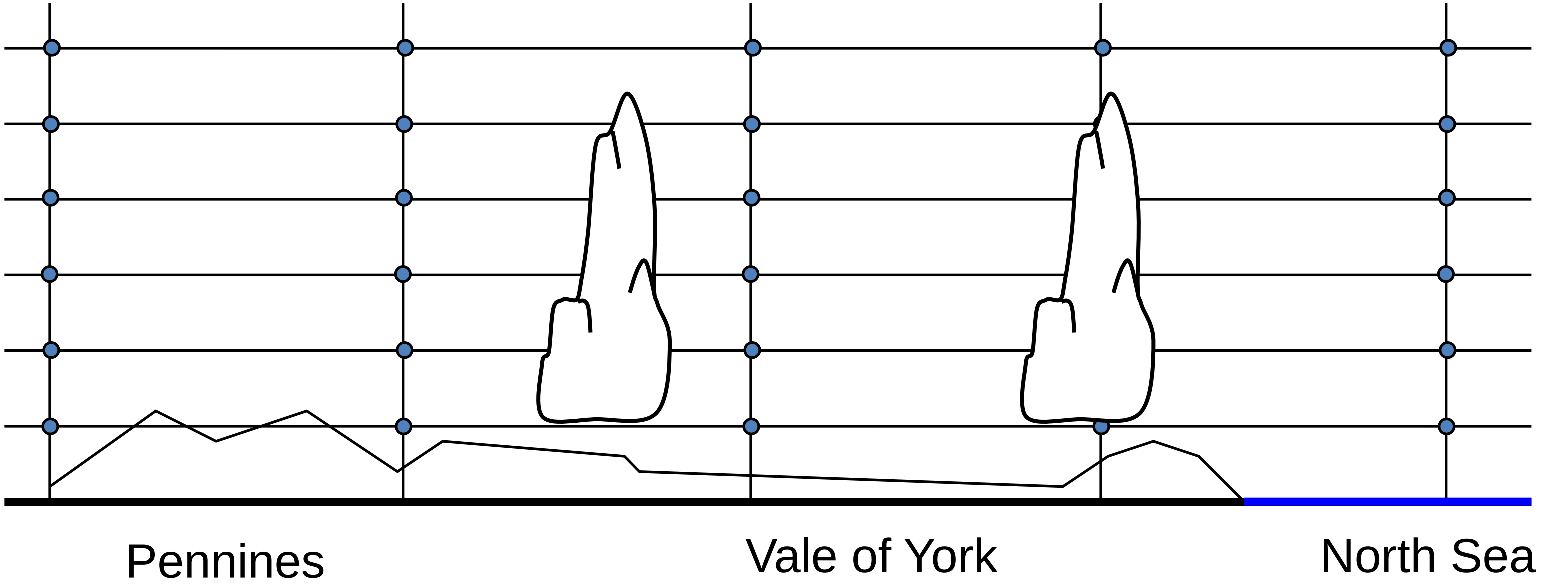
Parameterization

- **Parameterization** is the simplification of a complex physical process in terms of **parameters** that are available to the model, or readily measured.
- Models must use parameterizations of processes that :
  - Take place on scales smaller than the model grid
  - Involve parameters that are not explicitly defined in the model

Model resolution is too low to resolve:

- Individual clouds, even large thunderstorms.
- Full details of topography
- Details of changes of surface type

Processes on scales smaller than the grid must be **parameterized**.



Processes too small to resolve on grid:

- Computational expense limits resolution, Δx, of grid
- *Challenge:* some ("small-scale") phenomena of spatial scale, L, are too fine to be resolved and influence large-scale flow

$$L \ll \Delta x$$

  - convection, turbulent fluxes, radiation, cloud microphysics .

ODE for conserved qty, X, at a point : $DX/Dt = \Sigma_j S_{j,X}$

Processes too small to resolve on grid:

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- ***Approach:*** **estimate average net effects from unresolved processes in terms of resolved variables (parameterization)**
  - some unphysical assumptions needed (e.g. scale separation)
  - **parameterization** gives extra tendency, S_p, due to unresolved processes, after averaging ODE over grid-box (< >):

ODE solved on model grid: $D\langle X\rangle/Dt = \Sigma_i \langle S_{i,X}\rangle + S_p$

Microphysics param.

» Source and sink of vapour from clouds

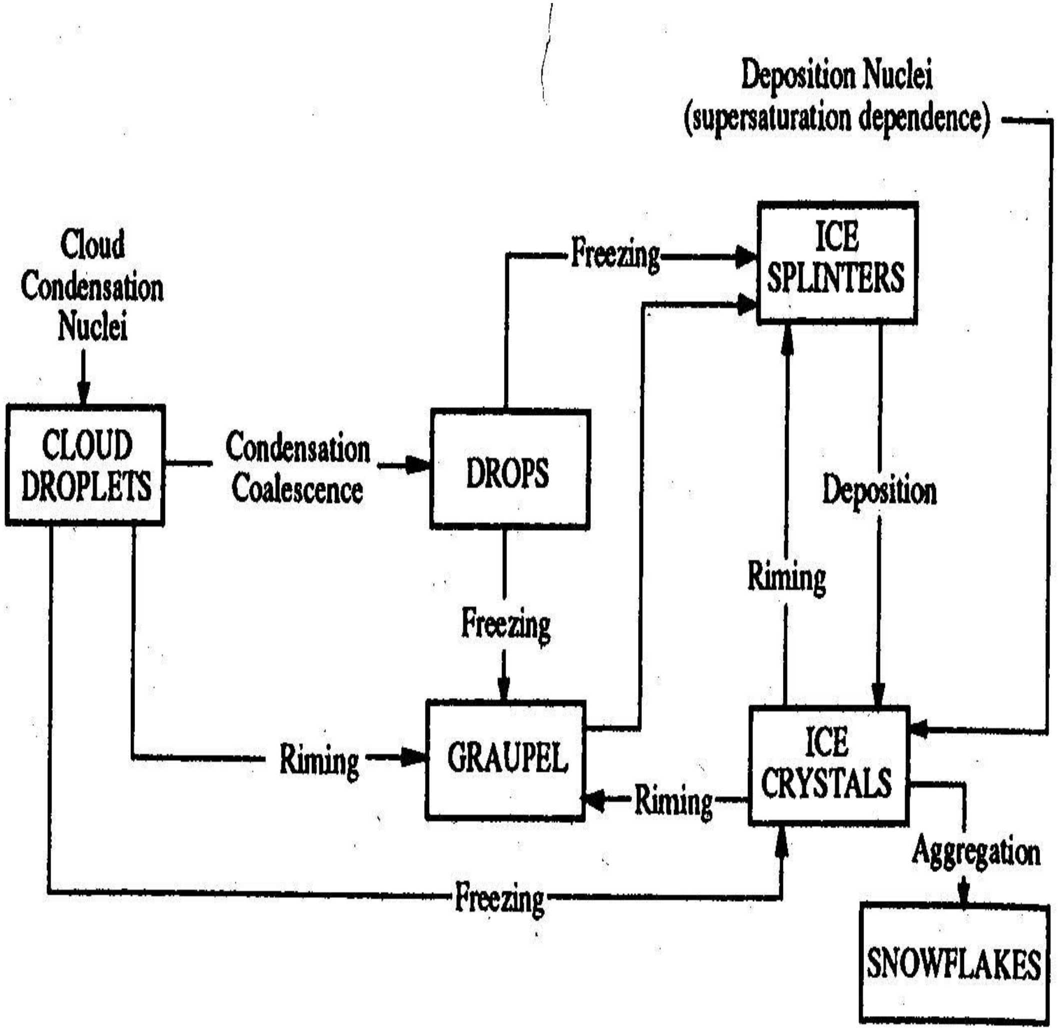
– Diffusion of vapour onto hydrometeors

» Condensation, vapour growth of ice, evaporation

– Coagulation

» Riming, ice-ice aggregation

» Coalescence



Turbulent mixing parameterisations

- PBL is characterised by turbulence
  - $X = X'(eddies) + \bar{X}\ \ (mean\ value)$
  - Turbulent eddies mix & homogenise (**diffusion**) heat .. ($\rho\ c_p\ \theta$), moisture ($\rho\ q_v$), momentum ($\rho\ u$), and their conserved variables, E.g. $X = \theta, q_v, u$
  - Flux is quantity per m^2 (normal to dirn.) per sec

- **Turbulent flux, f_X, of ρ X (in direction *s*), is proportional to gradient of mean of X**

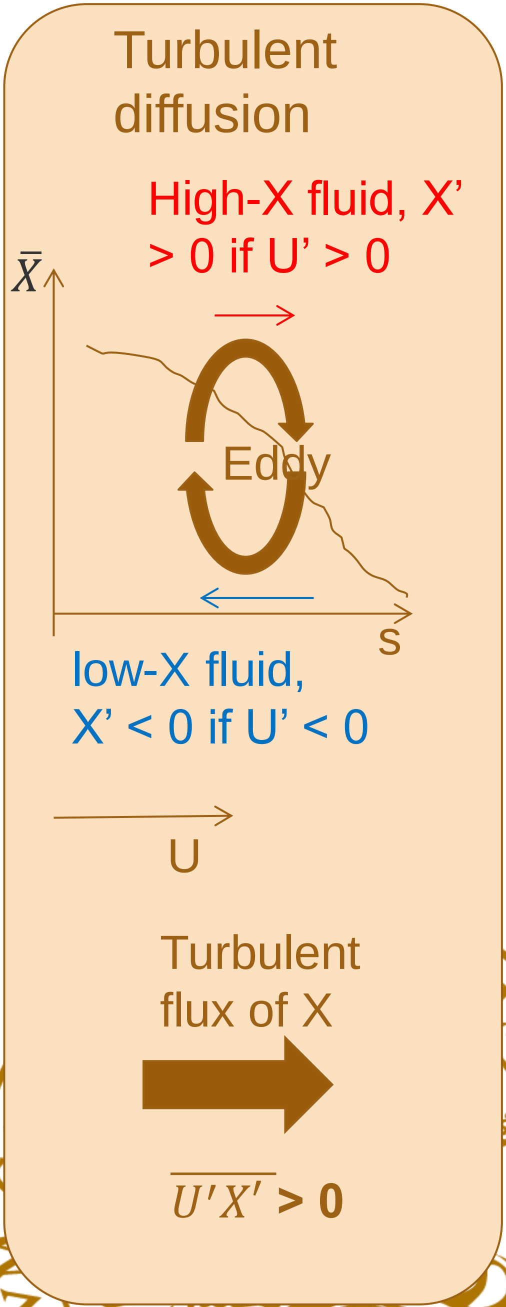
$$f_X = \rho\, \overline{U'X'} = -\rho K\, \frac{\partial \bar{X}}{\partial s}$$

  - $X = \theta$ or q_v in unsaturated parcels; $X = \theta_e$ or q_T in all (e.g. saturated) parcels; or else $X = u, v$ or w;
  - U is air speed in direction s, K is **turb. diffusivity**
    - U and s may be w and z for vertical mixing (PBL)
    - Or they could be u and x for horizontal mixing

- **Convergence of turbulent flux is source of the mean, $\bar{X}$, and the 1D diffusion eqn is:**

$$\frac{D\bar{X}}{Dt} \approx S - \frac{1}{\rho}\frac{\partial f_X}{\partial s} \approx S + K\frac{\partial^2 \bar{X}}{\partial s^2}$$

↖ Diffusion term



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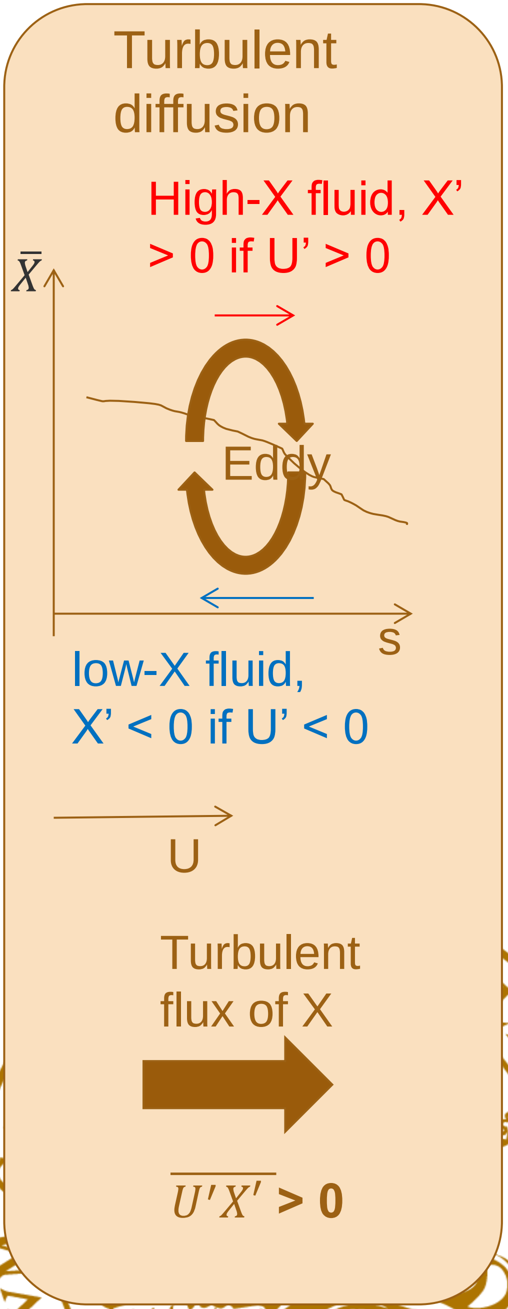

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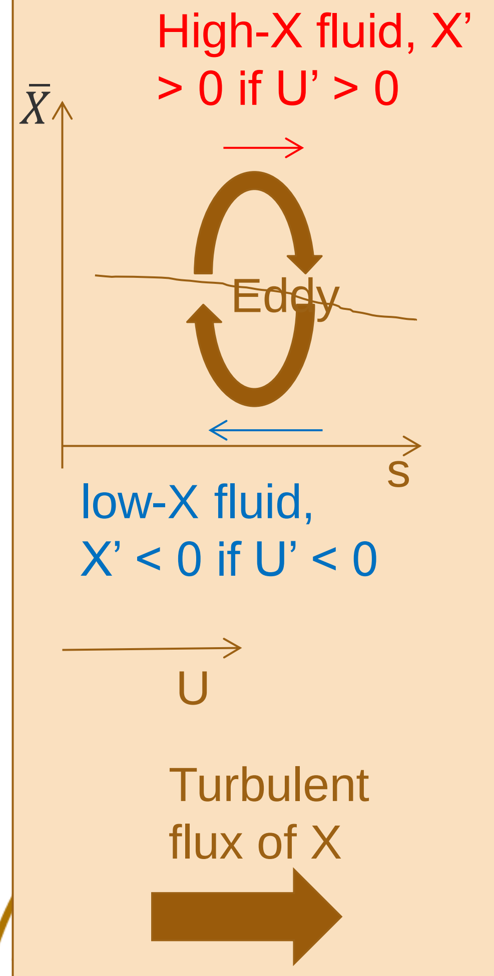
Turbulent diffusion



Turbulent flux of X

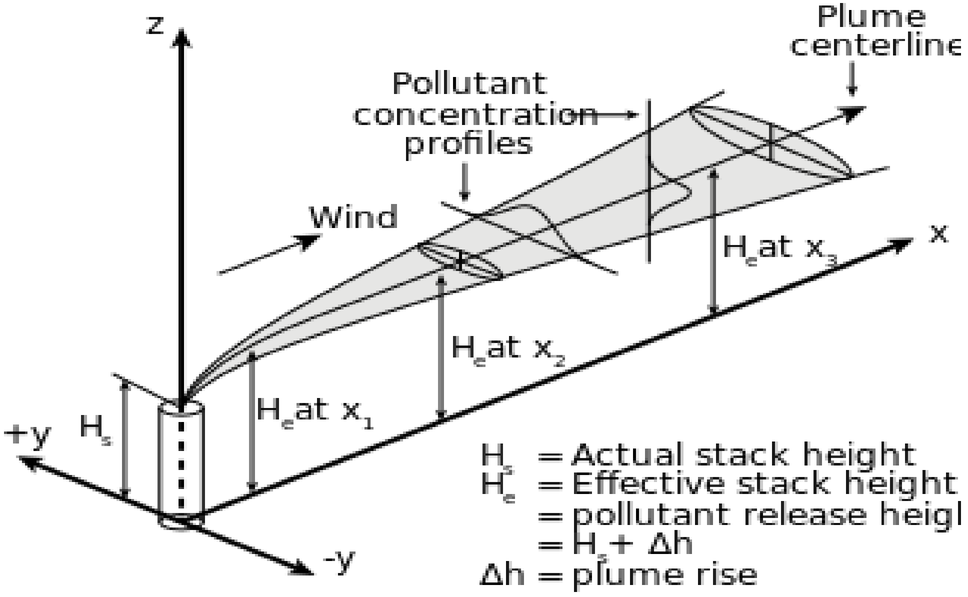
$\overline{U'X'} > 0$

Examples of diffusion in nature

» **Point-source of a tracer initially is diffused by 3D turbulence in PBL**

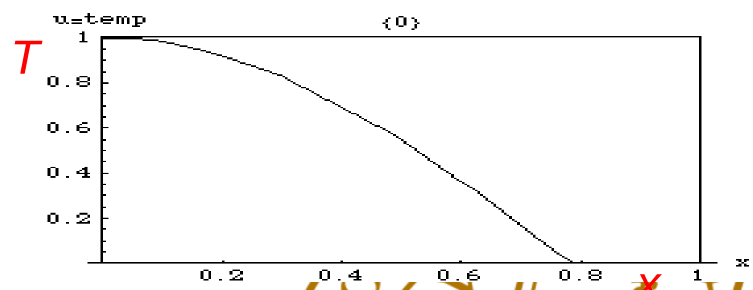
- Pollutant concentration, C, has an ever-widening Gaussian profile following the motion, until homogeneously mixed

$$\frac{DC}{Dt} = K\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + \;\ldots$$



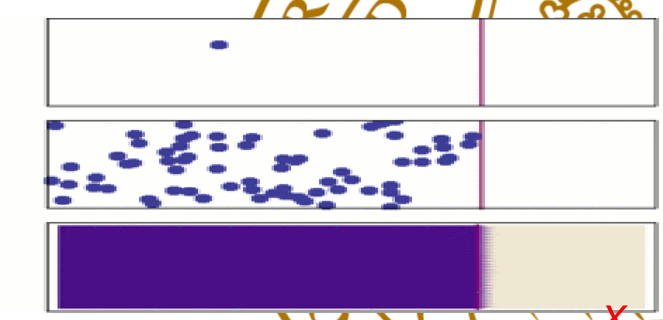
» **1D conduction or convection of heat in material with insulated ends:**

- $\frac{DT}{Dt} = K\frac{\partial^2 T}{\partial x^2}$



» **1D diffusion of solute in a fluid**

$$\frac{DC}{Dt} = K\frac{\partial^2 C}{\partial x^2}$$



SUMMARY

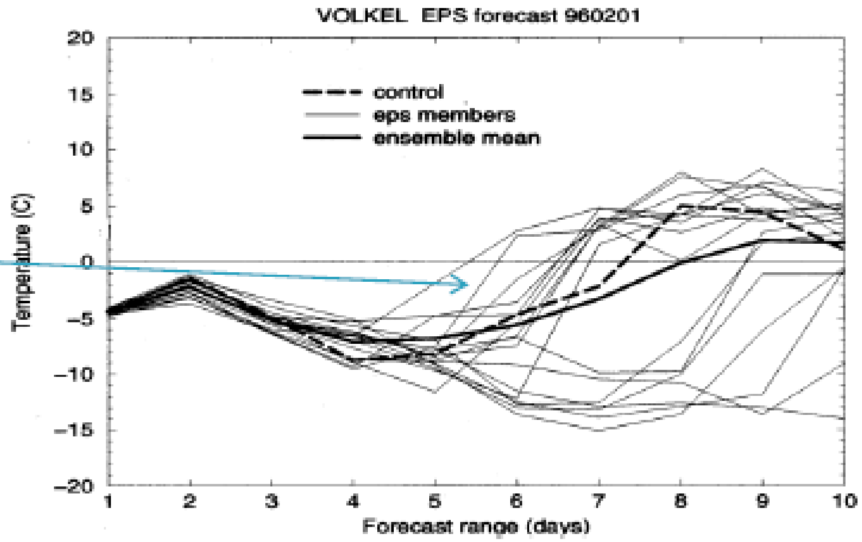
- Evolution equations of clouds are nonlinear
- Need to solve them numerically with finite-difference approximations for derivatives
- Model grid
- Sub-gridscale processes must be parameterized
  - Cloud microphysics
  - Turbulence
- Atmosphere is chaotic and difficult to predict
  - Especially for clouds !

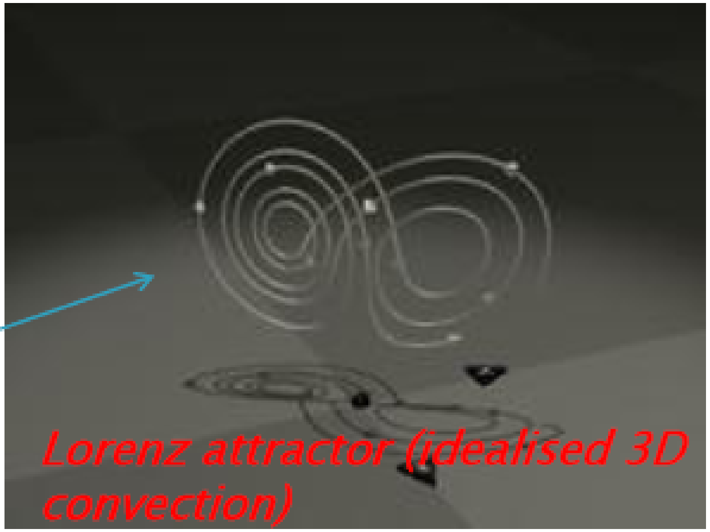
Obrigado

PREDICTABILITY AND CHAOS

Chaos

- **Chaotic systems** far from equilibrium have only **limited predictability** (e.g. atmosphere)
  - Small differences in initial conditions
    - big differences after a time
  - Predictability deteriorates with time, despite system being deterministic
- **Attractors** (sets of points in phase-space towards which system evolves)
  - order amid disorder





Lorenz attractor (idealised 3D convection)

Ensemble predictions

- Sensitivity to initial conditions = chaos
  - Atmosphere is chaotic
  - Errors in observing present atmosphere cause large forecast errors a few days ahead
  - Predictability depends on flow regime
- Ensemble of predictions differing in initial conditions is more accurate
  - Mean of ensemble usually more accurate than one of its members
  - Uncertainty in forecast and probability of different scenarios estimated