Introduction to Cloud Modeling. Part III: Convective Dynamics

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Outline

» Introduction

» Boussinesq Equations for Convective Scales

» Structure of Cell of Deep Convection

» Parcel Models of Dynamics of Convection

» Entrainment Simulated with Parcel Models

» Summary

» Further reading

INTRODUCTION

Scales and instabilities in atmosphere

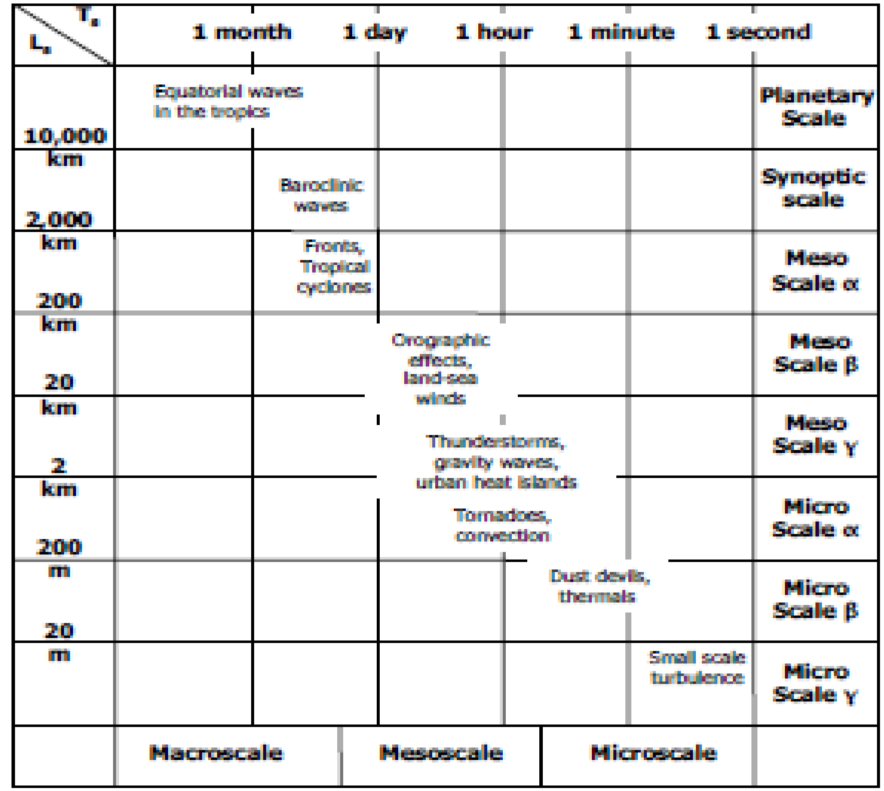
» Convective scales: L < 20 km ('L' or 'D' is horizontal width)

– Convective clouds

» Mesoscales: L < 2000 km

– Cloud systems (e.g. clusters convective cells)

» Synoptic scales: L > 2000 km



Major instabilities in atmosphere:

Type	Horizontal scale	Time scale	Destabilising force	Cloud-types
Conditional instability	$D \le 10$ km	$\frac{1}{N} \sim 8$ min	Buoyancy	CONVECTIVE
Symmetric instability	$\frac{(\partial u/\partial z)D}{f} \le 200$ km	$\frac{1}{f} \sim 3$ h	Coriolis	STRAT-IFORM
Baroclinic instability	$\frac{f^2(\partial u/\partial z)}{N^2\beta} \sim 2000$ km	$\frac{2\pi N}{(\partial u/\partial z)f} \sim 3$ days	Buoyancy / Coriolis	

Occurrence of thunderstorms:

» **Necessary factors for thunderstorms:**

- Some lifting mechanism to overcome CIN, triggering release of CAPE
- moisture – sufficient CAPE
- warmth in lower troposphere, coolness aloft - enough CAPE

» **Intensity and longevity of storms will be favoured by:**

- moderate wind shear in the background flow
- CAPE
  - » e.g. due to lack of previous deep convection

BOUSSINESQ EQUATIONS FOR CONVECTIVE SCALES

Frictionless equations of motion and heat

» Neglect Coriolis force and full equations become:

$$\frac{D\mathbf{v}}{Dt} = \mathbf{g} - \frac{1}{\rho}\nabla p$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla.\mathbf{v}$$

$$\boxed{\frac{D\theta}{Dt} = 0 \ \text{(unsat. adiabatic)} \ \text{ or } \ \frac{D\theta_e}{Dt} = 0 \ \text{(all adiabatic parcels)}}$$

» where $\mathbf{g} = (0,0,-g)$ and **v** = *(u,v,w)*

» For non-adiabatic motions (e.g. entrainment), S or S' replace zeros in thermodynamic equation

» Expression for $\boldsymbol{\theta_e}$ is derived thus:

Following the motion of a saturated (pseudo-)adiabatic parcel:

$$\frac{D\theta}{Dt} = -\frac{L}{c_p\Pi}\left(\frac{Dq_v}{Dt}\right)_{condensation} \Longrightarrow \frac{D\theta_e}{Dt} \approx 0$$

$$\Pi = T/\theta = (p/100000)^{R_d/c_p}$$

Boussinesq Equations of Motion

» Suppose pressure and density each have a hydrostatically balanced part that depends only on height:

$$p = \bar{p}(z) + p'(x, y, z, t)$$

$$\rho = \bar{\rho}\,(z) + \rho'(x, y, z, t)$$

$$\frac{\partial \bar{p}}{\partial z} = -\bar{\rho} g$$

T' is defined similarly. It follows that, if $\mathbf{F_B} = (0,0, F_B)$ and the pressure variable is $\phi = p/\bar{\rho}$

$$\frac{D\mathbf{v}}{Dt} \approx \mathbf{F_B} - \underbrace{\frac{1}{\bar{\rho}}\boldsymbol{\nabla} p'}_{\text{PPGF}} = \mathbf{F_B} - \underbrace{\boldsymbol{\nabla}\phi'}_{\text{PPGF}}$$

$$F_B = -\frac{g\rho'}{\bar{\rho}} \approx \frac{gT'}{\bar{T}} \approx \frac{g\theta'}{\bar{\theta}}$$

Boussinesq Equations of Continuity: assume no sound waves ($\rho' = 0$)

» Thermodynamic eqn as above

» Shallow atmosphere: incompressibility following the motion

$$\frac{1}{\rho}\frac{D\boldsymbol{\rho}}{Dt} = 0 \Rightarrow \boxed{\boldsymbol{\nabla}.\,\mathbf{v} = 0}$$

» Deep atmosphere: only when horizontal is motion incompressible

$$\boxed{\boldsymbol{\nabla}.\left(\bar{\rho}(z)\mathbf{v}\right) = 0}$$

» Bousinesq equations allow gravity waves but not sound waves

» **Boussinesq approximation: density is treated as constant at each level, except where it appears in buoyancy force**

Optional extra approximations for Boussinesq Eqns

» Atmosphere assumed adiabatic for idealised studies, with constant potential temperature, θ_0, at all levels:

$$\phi = \frac{p}{\bar{\rho}} = c_p \theta_0 \left(\frac{p}{p_0}\right)^{\kappa} \qquad \bar{\rho}\,(z) = \bar{\rho}\,(0)\; fnc(\,z, \theta_0)$$

» For simplest idealised model, neglect vertical PPGF with parcel assumption, neglect friction, assume (saturated) unsaturated adiabatic parcels:

» $\frac{Du}{Dt} \approx -\frac{\partial \phi'}{\partial x}$

» $\frac{Dv}{Dt} \approx -\frac{\partial \phi'}{\partial y}$

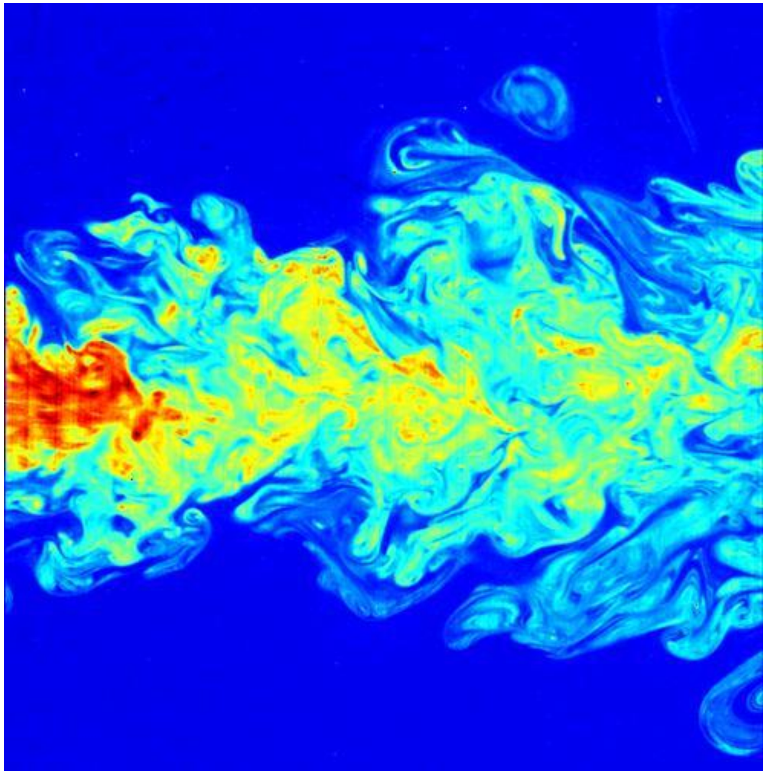
» $\frac{Dw}{Dt} \approx F_B \approx \frac{g\theta'}{\bar{\theta}}$

$$\frac{D\theta_{(e)}}{Dt} \approx 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

BOUNDARY LAYER: DRY CONVECTION, TURBULENCE AND SHEAR

Turbulence



- » Turbulence = random, rotational motion of fluid (when $Re = U L / \nu >$ critical threshold)
  - – variability on many scales
- » Planetary Boundary Layer (PBL) is part (e.g. lowest ~1 km) of troposphere affected by Earth's surface by turbulent fluxes
- » Turbulence characterises PBL, where low convective clouds have bases
- » Turbulence consists of eddies, which in 3D are vortex tubes
  - – Tubes form closed loops or terminate on ground (e.g. tornado)

Turbulence

» Turbulent eddies in atmosphere are on scales less than 100s of metres

– Unresolved mostly by atmospheric models

» KE of turbulence (‘TKE’) transfers between eddies on different spatial scales

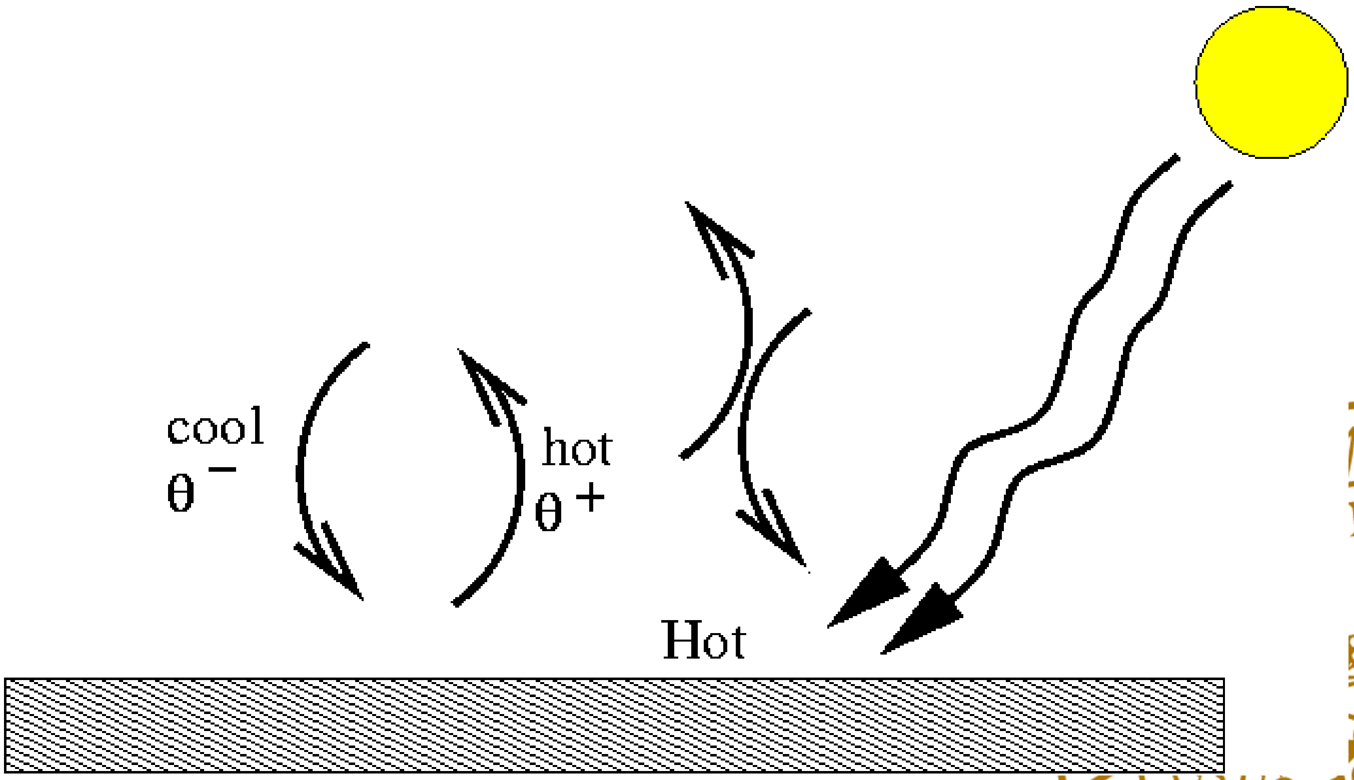
– In 3D, cascade of TKE to smaller and smaller scales from the forcing scale, until becoming heat

Importance of turbulence

- » Turbulence is important in mixing and transport.
- » Inside PBL, turbulence transports heat, moisture and other trace gases away from / towards the Earth's surface.
  - – Determines humidity and cloud formation.
- » Turbulent motions in PBL (e.g. from surface heating) may trigger convective clouds.
- » Turbulent mixing determines properties of deep convective clouds, by diluting cloudy air with environmental air.

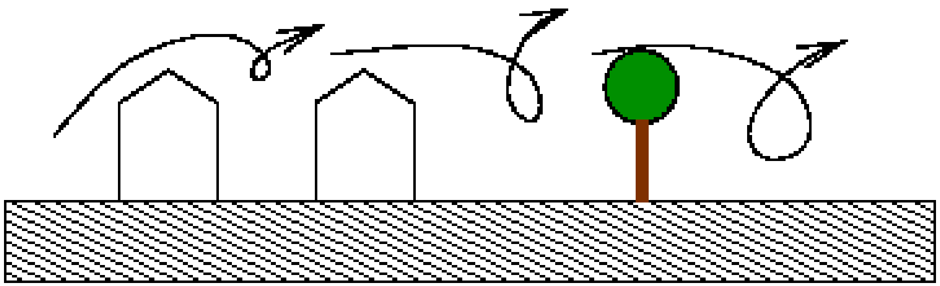
Sources of turbulence

» Buoyant convection

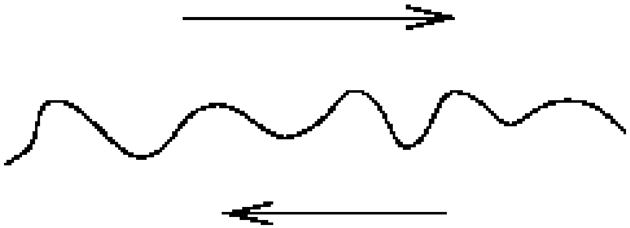


Sources of turbulence

» Mechanical or shear-driven turbulence

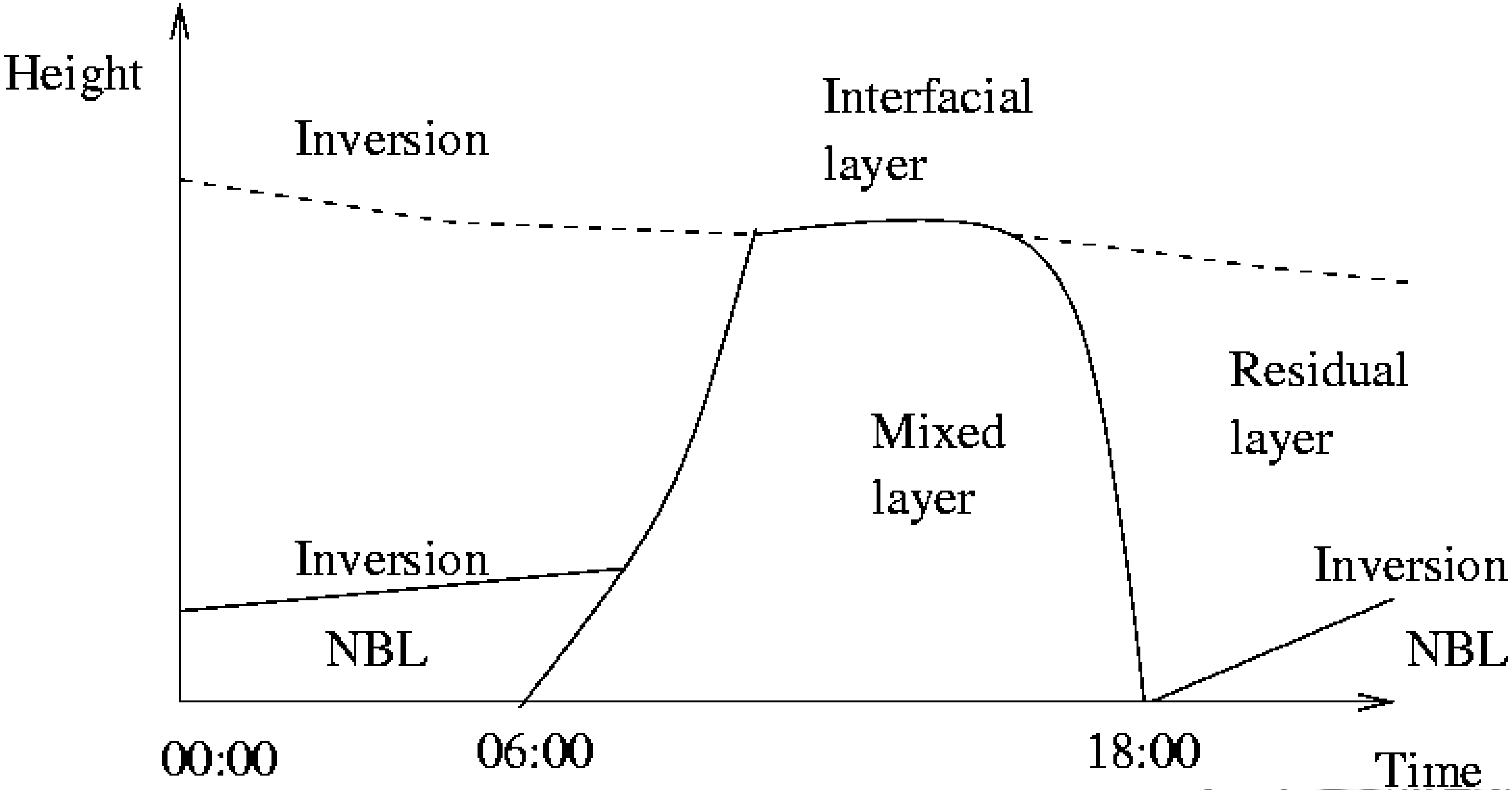


At Earth's surface



Aloft

Turbulence and stability - diurnal cycle of PBL



Simplest model of turbulence

» turbulent fluxes depend on intensity of turbulence (turbulent diffusivity, *K*) and the gradient of the mean of quantity transported

vertical flux of horizontal momentum, $f_x = \rho\,\overline{u'w'}$

$$= -\rho K \,\frac{\partial \bar{u}}{\partial z}$$

» 'Friction' from turbulent mixing is opposite to direction of motion

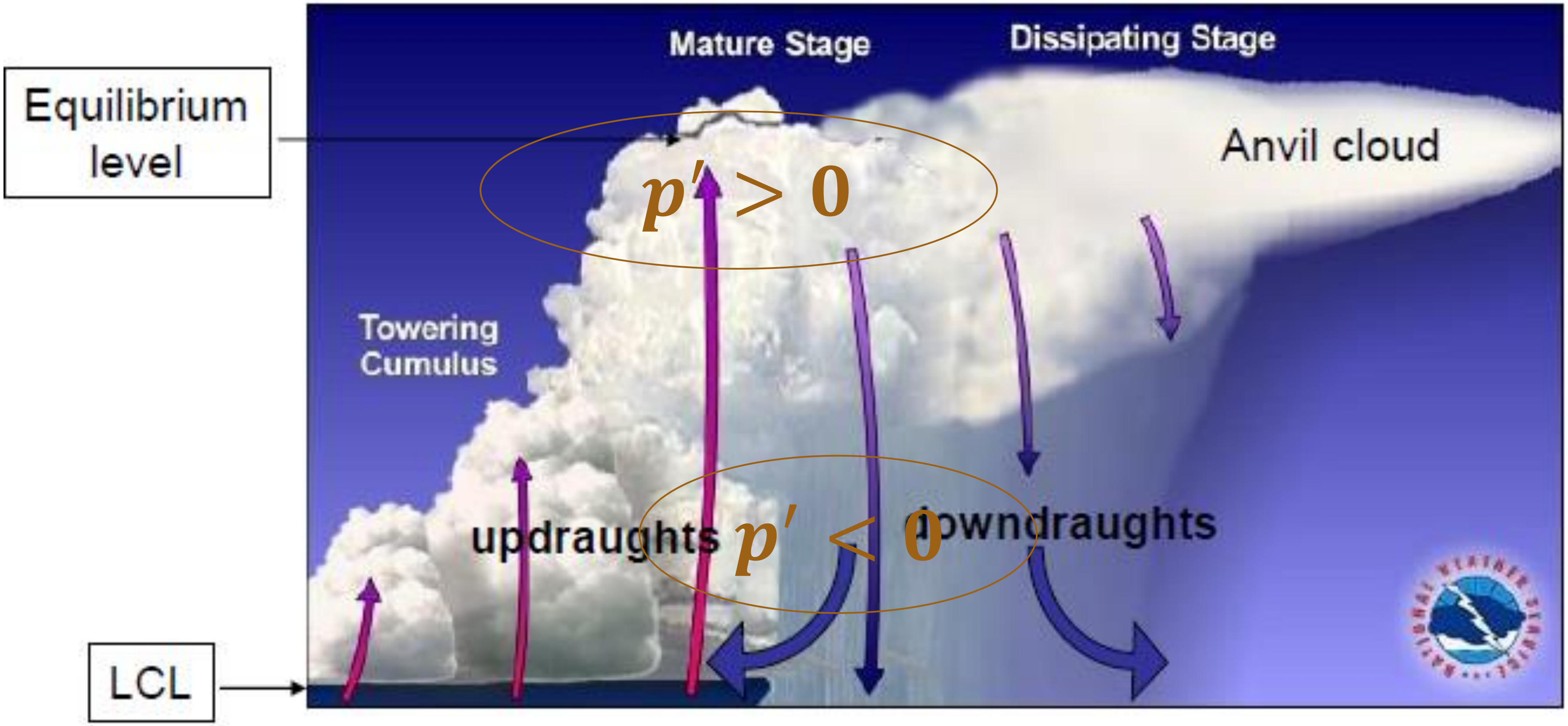
$$\frac{D\bar{u}}{Dt} \approx -\frac{\partial \phi'}{\partial x} - \frac{1}{\rho}\frac{\partial f_x}{\partial z}$$

» *in the surface layer* (the lowest 10% of the boundary layer, where TKE is generated), momentum flux is constant over height ($-\overline{u'w'} = u_*^2$ and $\mathrm{K} = \rho\,(kz)^2\,\left|\frac{\partial \bar{u}}{\partial z}\right|$) for near-neutral flow and:

$$\bar{u} = (u_*/k)\,ln(z/z_0)$$

PARCEL MODELS OF DYNAMICS OF CONVECTION

Structure of deep convective cell (e.g. Cb thunderstorm)



Cold pool and gust front

» Downdrafts are caused by evaporation of rain, gravitational burden of precipitation, and downward PPGF from ascent

» Once precipitation starts to fall, cold downdraughts descend to the surface, where a **cold pool** spreads out below cloud.

» Edge of cold pool is **gust front,** where

– Horizontal gradient in buoyancy generates vorticity and turbulence

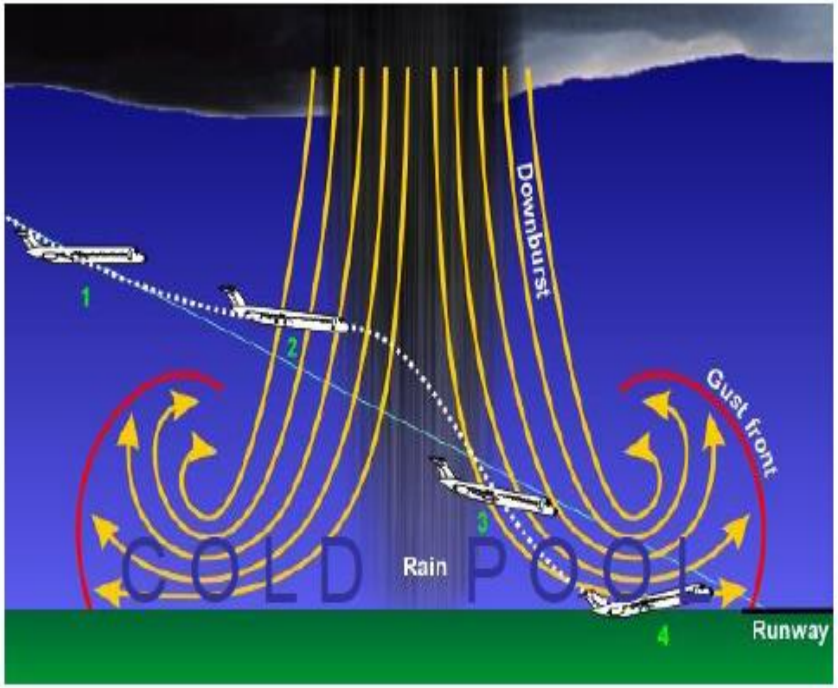
$$\frac{D\theta}{Dt} = -\frac{L}{c_p\Pi}\left(\frac{Dq_v}{Dt}\right)$$

Tendency in downdraft from evaporation of rain due to RH < 100%, reducing F_B

$$F_B = g\left[\frac{(\theta_{v,parcel} - \theta_{v,env})}{\theta_{v,env}} - q_H\right]$$

Gravitational burden of rain in downdraft and PPGF

$$\boxed{\frac{Dw}{Dt} \approx F_B - \frac{\partial\phi'}{\partial z}}$$



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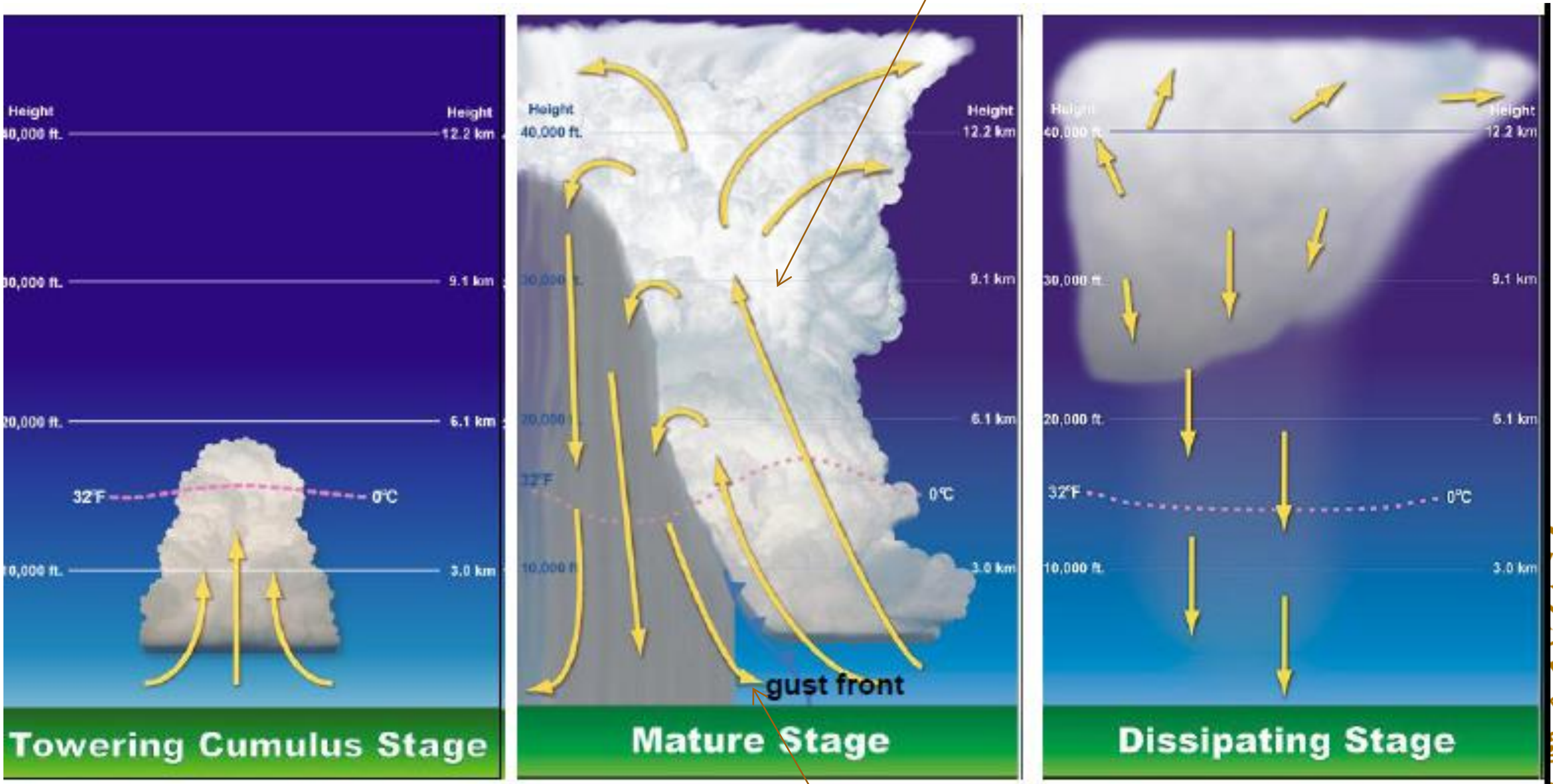
Life-cycle of single cell

Ascent in updraft
condensation
precipitation



Cell will decay when cold pool spreads so far that it cuts off the inflow of warm moist air to updraft

ENTRAINMENT SIMULATED WITH PARCEL MODELS

» **To define CAPE, LFC, LCL … we assumed parcels are adiabatic:**

– Equivalent potential temperature, $\theta_{e,parcel}$ = constant

– Total water mixing ratio (vap+liq), $Q_{T,parcel}$ = constant

(sat or unsat) adiabatic parcels

» **But in reality, convection creates turbulence in a ‘cascade’ of KE:**

– ‘Entrainment’: Turbulent mixing dilutes cloudy parcels with cold dry environmental air

» reduces F_B, lowers level of neutral buoyancy below undilute EL

» 1D model of continuous entrainment at rate, *E = (Dm/Dt)/m*:

» $\frac{D\theta_{e,parcel}}{Dt} = -E\ (\theta_{e,parcel} - \theta_{e,env})$

» $\frac{DQ_{T,parcel}}{Dt} = -E\ (Q_{T,parcel} - Q_{T,env})$

model of real parcels

- **Observations: highest cloud-top level is close to undilute EL as mixing is probabilistic and occurs in discrete mixing events**
  - a *few* parcels ($E \approx 0$) are always un-mixed and detrain at undilute EL, most are mixed ($E > 0$) detrain at lower levels

SUMMARY

- Turbulence generated by buoyancy gradients and shear
- PBL depth cycles between minimum (night) and maximum (day)
  - steepening of lapse rate in PBL from surface heating drives convection, which drives turbulence
- Turbulence causes mixing, diluting most (but not all) cloudy updraft parcels and reducing their buoyancy
  - Continuous 1D entrainment model if mixing is rapid
- Below clouds: cold pool of downdraft air, driven by precipitation
  - Turbulence at gust front
  - Cold pool spreads out, eventually causing decay of cell
- Triggers of convection: forced ascent of air to saturation and LFC
  - convective clouds usually have bases in lower troposphere
  - parcels from too high in troposphere will not have an LFC

Obrigado

Perturbation-pressure gradient force (PPGF)

- **As parcel ascends due to buoyancy force, surrounding air must move out of the way above and fill its wake below**
  - High and low pressure perturbations above and below it
  - PPGF = $-\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} < 0$, so is directed downward
- ***<u>Thought experiment</u>***:
  - if width of parcel increases, the perturbations also increase until PPGF equals buoyancy force and then always $\frac{Dw}{Dt} \approx 0$
    - Hydrostatic balance
  - if width goes to zero (e.g. in convection), PPGF goes to zero
    - No hydrostatic balance (‘non-hydrostatic’ motions)
    - buoyancy causes $\frac{Dw}{Dt} \neq 0$

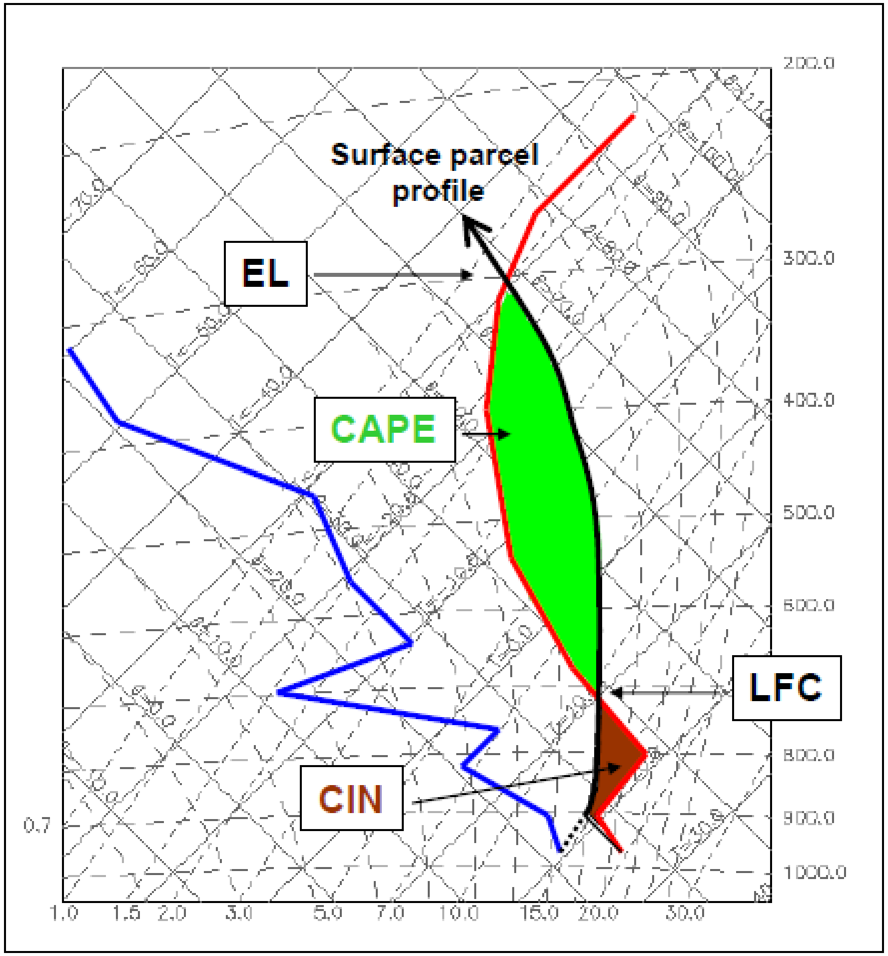
CI measured by CAPE – But often CAPE is large and no convection occurs

» CAPE is energy of environment available to near-surface parcels to convert to their own kinetic energy (vertical motion) once they have reached LFC.

  – The greater the CAPE, the stronger any convection and storms are likely to be.

» But if CIN is too strong, or if there is too little lifting of parcels, release of CAPE (even if large) will not happen

  – Convective clouds unlikely

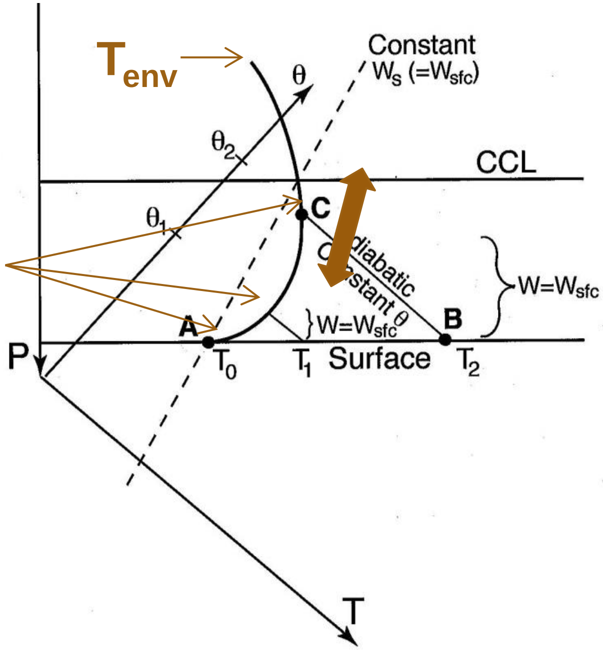


Diurnal cycle of continental PBL

- **Daytime convective boundary layer (mostly unsaturated): turbulence is generated by buoyant convection**.
  - PBL deepens as ground warms in sunlight, warming sfc air
  - Clouds may form if LCL of sfc air (near CCL) is reached by PBL top

- **Night-time stable boundary layer: stability suppresses convective vertical motions and so reduces turbulence**.
  - PBL collapses almost to surface, ground cools sfc air (e.g. dew)
  - Only mechanical generation of turbulence

Top of PBL at various times during day and night



BC = daytime profile of T_{env} in PBL

AC = night-time profile in free/troposphere, no turbulent PBL