Atmospheric Aerosol Physics, Physical Measurements, and Sampling

Electrical Mobility & Differential Mobility Analyzer

São Paulo School of Advanced Science on Atmospheric Aerosols:
properties, measurements, modeling, and effects on climate and health

Member of Leibniz Association

ECAC
European Center for Aerosol Calibration

WCCAP
World Calibration Centre
for Aerosol Physics

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Leibniz Institute for
Tropospheric Research

Electrical Particle Mobility

Force in an Electrical Field

The electrical force in an electrical field can be calculated to

$$\boxed{\vec{F}_{\mathrm{el}} = n \cdot e \cdot \vec{E}}$$

with:

e … elemental charge 1,602·10^{-19} As

n … number of charges

Electrical Particle Mobility

In equilibrium (the electrical force equals the drag force), the resulting velocity can be calculated analog to the sedimentation velocity.

$$\boxed{\vec{F}_D = \vec{F}_{el}}$$

$$\boxed{\vec{u}_e = \vec{F}_{el} \cdot B = n_e \cdot \mathrm{e} \cdot \vec{E} \cdot B = n_e \cdot \mathrm{e} \cdot \vec{E} \cdot \frac{C_C}{3\pi \cdot \eta \cdot D_\mathrm{P}}}$$

The mobility of aerosol particles in an electrostatic field is called electrical particle mobility.

The electrical mobility Z_P of a particle with a certain electric charge is defined to:

$$\boxed{Z_\mathrm{P} = \frac{\vec{u}_e}{\vec{E}} = n_e \cdot \mathrm{e} \cdot \frac{C_C}{3\pi \cdot \eta \cdot D_\mathrm{P}}}$$

The electrical mobility is given in [cm^2/Vs].

The relation between the electrical and mechanical mobility is given to:

$$\boxed{Z_{\mathrm{P}} = n_{\mathrm{e}} \cdot \mathrm{e} \cdot B}$$

The relation between the electrical mobility and the diffusion coefficient is described by:

$$\boxed{Z_{\mathrm{P}} = \frac{n_{\mathrm{e}} \cdot \mathrm{e}}{k \cdot T} \cdot D}$$

Movement in an Electrical Field & Plate Mobility Analyzer

Assumption

- The electrical mobility is than only a function of the particle size and electric charge.
- The flow is constant over the entire width of the plate capacitor.

Example

- An electrically charged polydisperse aerosol is led through a plate capacitor.
- Electrically charged particles are separated and deposited according their size.

Plate capacitor

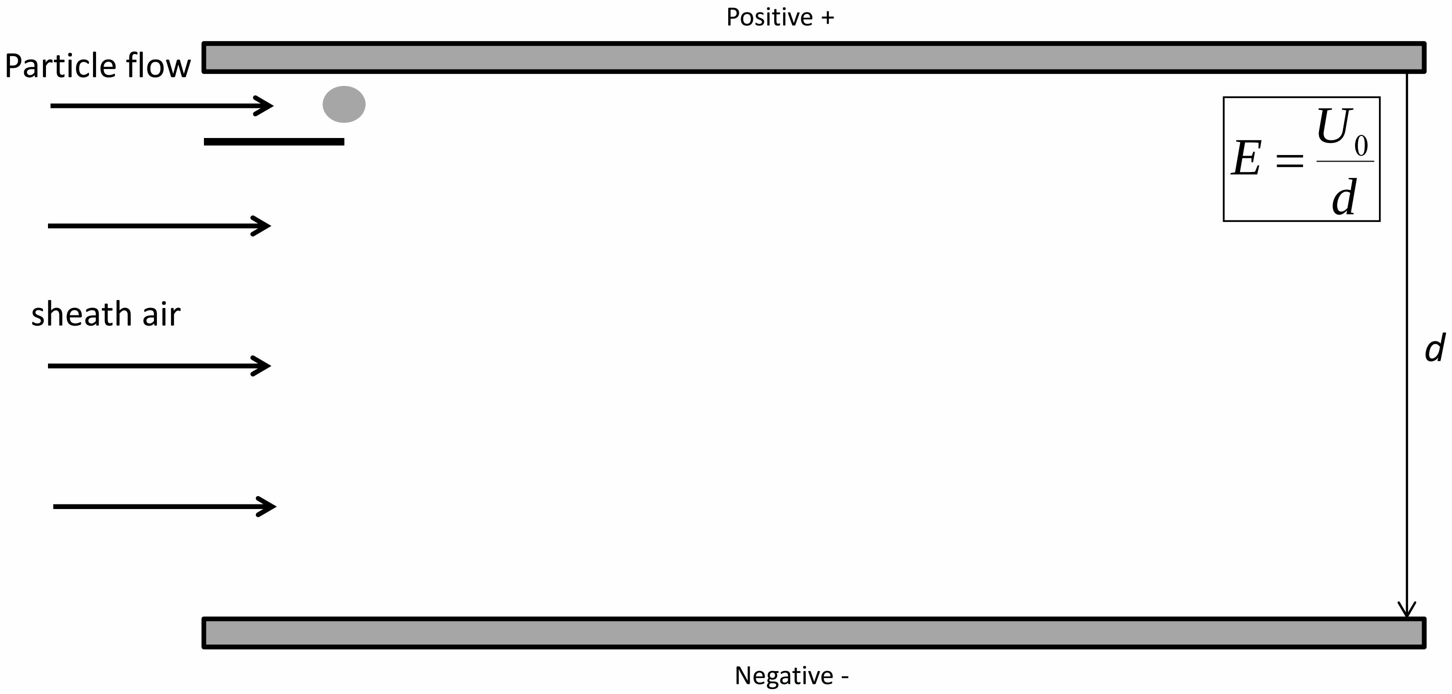
Voltage between plate 1 and plate 2: U_0

The potential decreases linearly from plate 1 to plate 2.

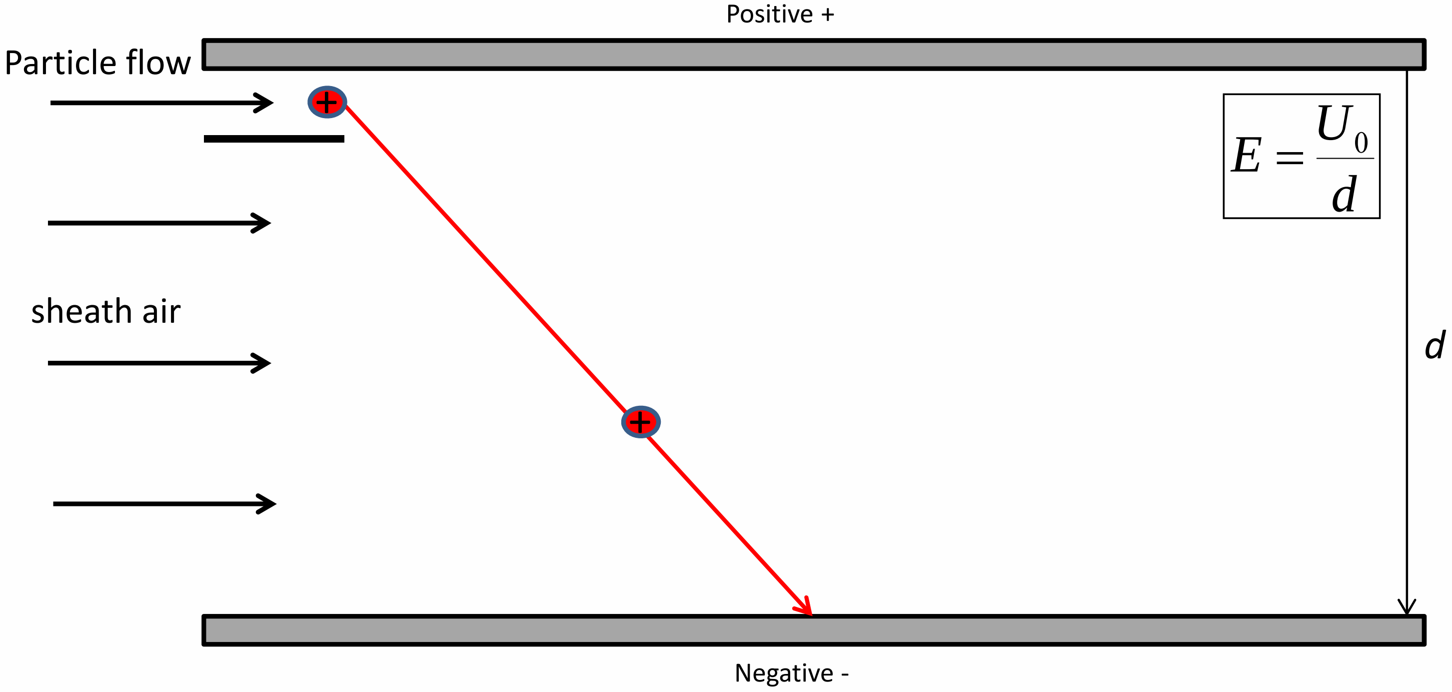
The electric field between the plates is homogenous.

$$\boxed{E = \frac{U_0}{d}}$$

d … distance between the plates



$$E = \frac{U_0}{d}$$



Positive +

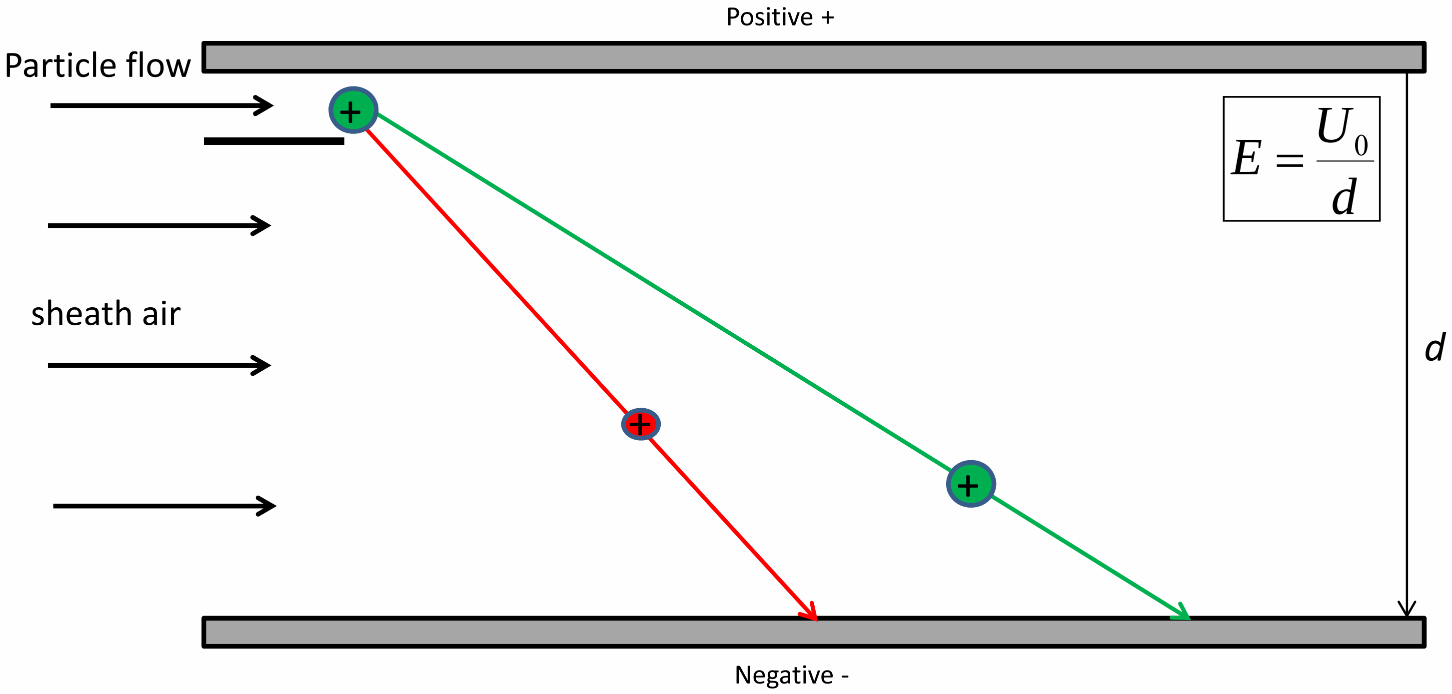
Particle flow

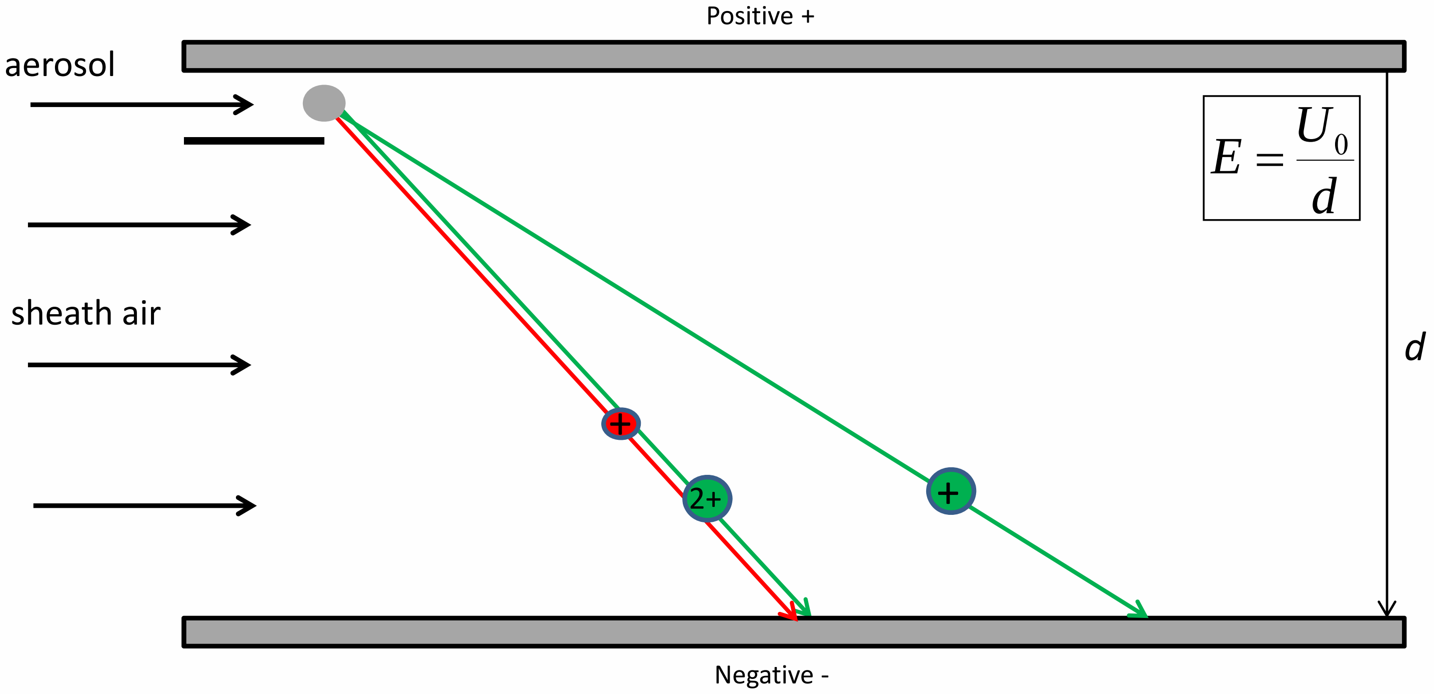
sheath air

$$E = \frac{U_0}{d}$$

d

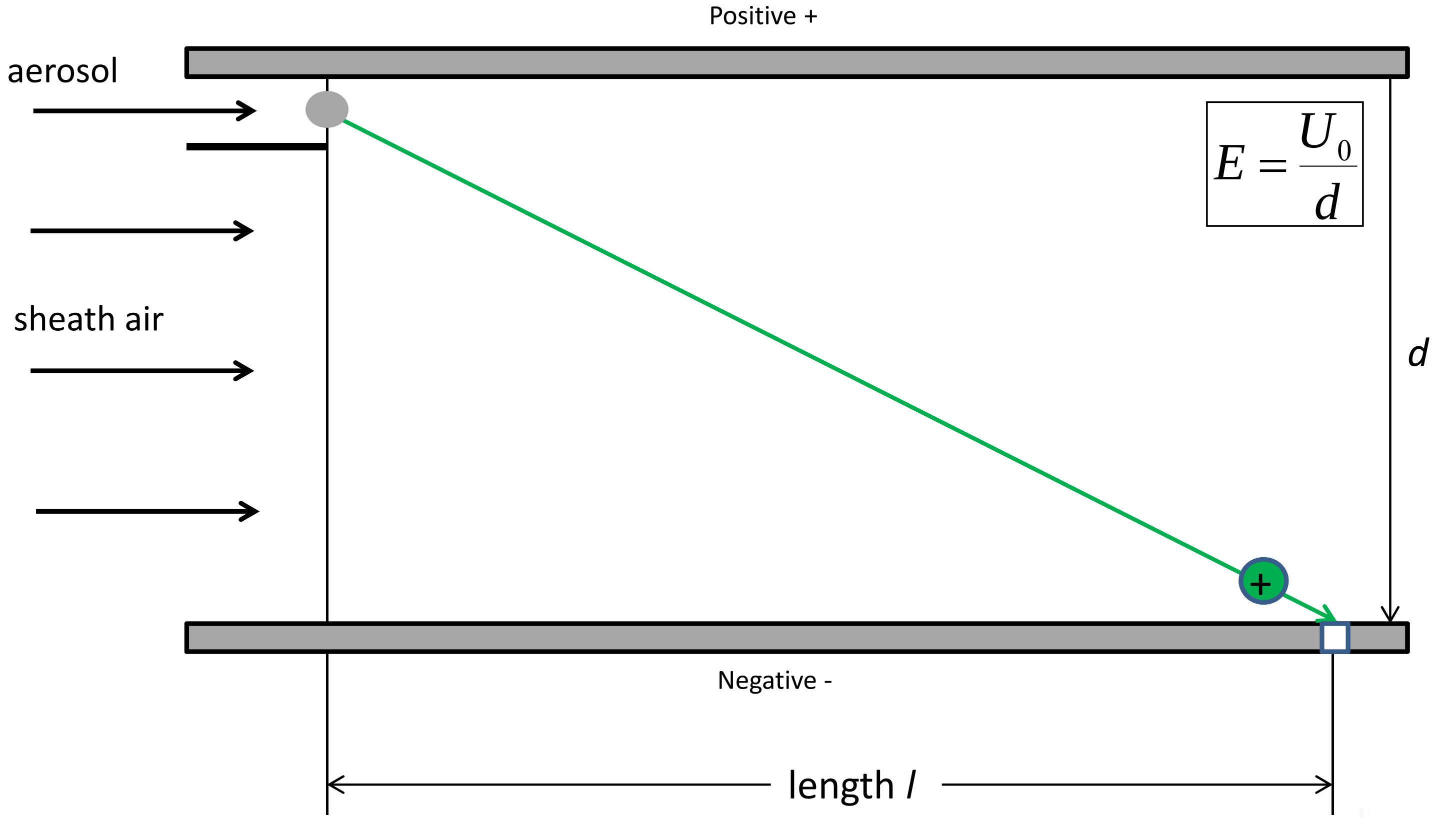
Negative -


Positive +
Particle flow
sheath air
$E = \frac{U_0}{d}$
d
Negative -



$$E = \frac{U_0}{d}$$

Plate differential mobility analyzer



$$E = \frac{U_0}{d}$$

Theory of a Differential Mobility Analyzer-

Plate DMA

- An electric field is put between the plates (z-direction).
- A laminar particle-free sheath air flow Q_{sh} is led through the capacitor (x-direction).
- The aerosol flow Q_A (x-direction) is fed into the capacitor close to one plate.

The total volume flow is

$$\boxed{Q = Q_{Sh} + Q_A}$$

The particle velocity in x-direction is given to:

$$\boxed{u_x(z) = \frac{\mathrm{d}x}{\mathrm{d}t}}$$

$$\boxed{\mathrm{d}x = u_x(z) \cdot \mathrm{d}t}$$

The particle velocity in z-direction is defined to:

$$\boxed{u_\mathrm{z} = \frac{\mathrm{dz}}{\mathrm{d}t} = E_z \cdot Z_\mathrm{P} = \frac{U \cdot Z_\mathrm{P}}{d}}$$

➔ $$\boxed{\mathrm{d}t = \frac{d}{U \cdot Z_\mathrm{P}}\,\mathrm{dz}}$$

➔ $$\boxed{\mathrm{d}x = \frac{d \cdot u_x(\mathrm{z})}{U \cdot Z_\mathrm{P}}\,\mathrm{dz}}$$

➔ $$\boxed{\int_0^{\mathrm{l}} \mathrm{d}x = \frac{d}{U \cdot Z_\mathrm{P}} \int_0^{\mathrm{d}} u_x(\mathrm{z})\,\mathrm{dz}}$$

with $$\boxed{\bar{u}_x = \frac{Q}{w \cdot d}}$$ $$\boxed{l = \frac{Q \cdot d}{w \cdot U \cdot Z_\mathrm{P}}}$$

w … width of the capacitor

d … distance between plates

The maximum electrical mobility for a certain deposition place is given to:

$$\boxed{Z_\mathrm{P} = \frac{Q \cdot \mathrm{d}}{\mathrm{w} \cdot U \cdot \mathrm{l}}}$$

The voltage to select a certain mobility can be calculated by:

$$\boxed{U = \frac{Q \cdot \mathrm{d}}{\mathrm{w} \cdot l \cdot Z_\mathrm{P}} = \frac{Q \cdot \mathrm{d}}{\mathrm{w} \cdot \mathrm{l}} \cdot \frac{3\pi \cdot \eta \cdot D_\mathrm{P}}{n_e \cdot \mathrm{e} \cdot C_\mathrm{C}(D_\mathrm{P})}}$$

Cylindrical DMA

$$\boxed{E = \frac{U_0}{\ln(r_o/r_i)\cdot r}}$$

r … radial position
r_i … radius of the inner electrode
r_o … radius of the outer electrode

Cylindrical capacitors are used in modern aerosol instrumentations.

The total flow is given to:

$$\boxed{Q = Q_{Sh} + Q_A}$$

The particle velocity in x-direction is given to:

$$\boxed{u_\mathrm{x}(r) = \frac{\mathrm{d}x}{\mathrm{d}t}}$$

➔ $$\boxed{\mathrm{d}x = u_\mathrm{x}(r) \cdot \mathrm{d}t}$$

The radial velocity due to the electric field is described to:

$$\boxed{u_r(r) = \frac{\mathrm{d}r}{\mathrm{d}t} = E_r \cdot Z_\mathrm{P} = \frac{U \cdot Z_\mathrm{P}}{r \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}}$$

➔ $$\boxed{\mathrm{d}t = \frac{r \cdot \ln(\mathrm{r_a}/\mathrm{r_i})}{U \cdot Z_\mathrm{P}} \mathrm{d}r}$$

with $$\boxed{\overline{u}_x = \frac{Q}{\pi \cdot (\mathrm{r_o^2} \text{-} \mathrm{r_i^2})}}$$

➔ $$\boxed{\mathrm{d}x = \frac{r \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{U \cdot Z_\mathrm{P}} u_\mathrm{x}(r) \mathrm{d}r}$$

➔ $$\int_0^1 \mathrm{d}x = \frac{\ln(\mathrm{r_o}/\mathrm{r_i})}{U \cdot Z_\mathrm{P}} \int_{\mathrm{r_i}}^{\mathrm{r_a}} r \cdot u_x(r)\mathrm{d}r = \frac{\overline{u}_x \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{U \cdot Z_\mathrm{P}} \int_{\mathrm{r_i}}^{\mathrm{r_a}} r \cdot \mathrm{d}r$$

➔ $$l = \frac{u_x \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2 \cdot U \cdot Z_\mathrm{P}} (\mathrm{r_o}^2 - \mathrm{r_i}^2) = \frac{Q \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot Z_\mathrm{P}}$$

The maximum electrical particle mobility is given to:

$$Z_\mathrm{P} = \frac{Q \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}$$

The voltage to select a the maximum electrical particle mobility can be calculated to:

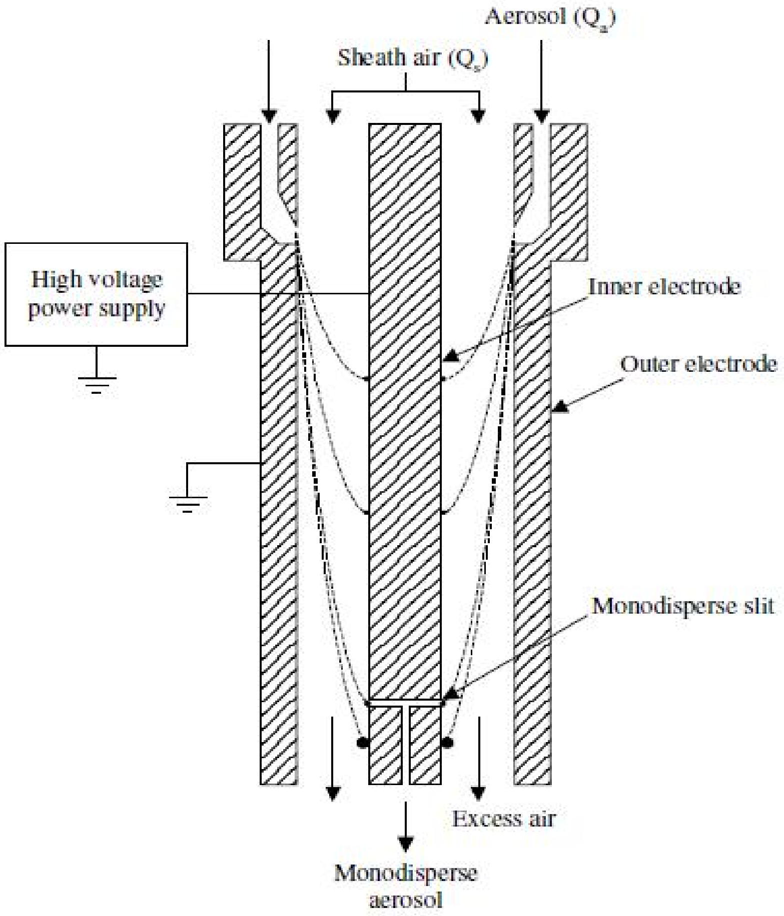
$$\boxed{\frac{n_e \cdot \mathrm{e} \cdot C_\mathrm{C}}{3\pi \cdot \eta \cdot D_\mathrm{P}} = \frac{Q \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}}$$

$$\Rightarrow \qquad \boxed{U = \frac{3\eta \cdot D_\mathrm{P} \cdot Q \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2 \cdot 1 \cdot n \cdot \mathrm{e} \cdot C_\mathrm{C}(D_\mathrm{P})}}$$

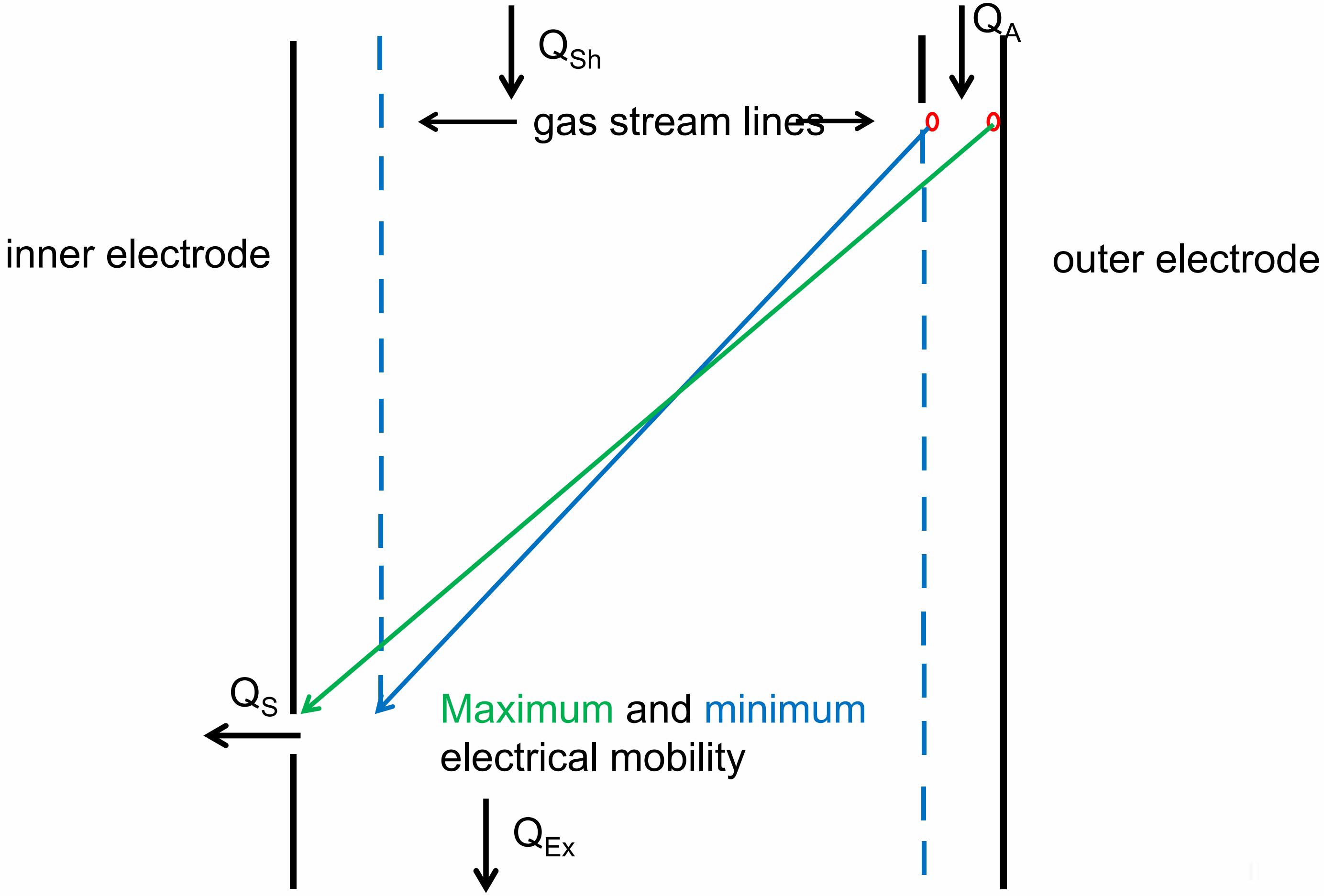
The maximum particle diameter can be calculated as follows, but it cannot be analytically solved:

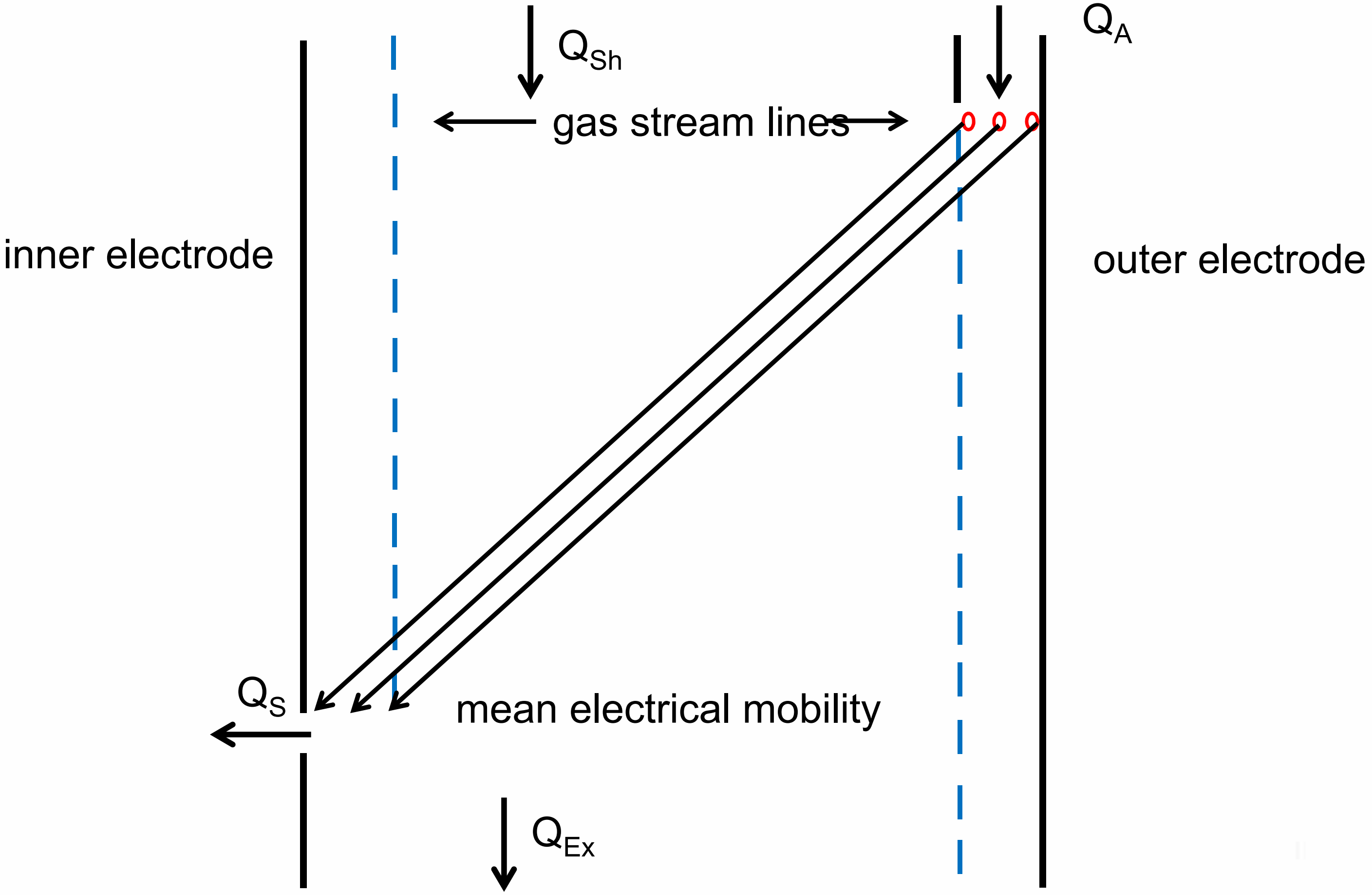
$$\Rightarrow \qquad \boxed{D_\mathrm{P} = \frac{2 \cdot U \cdot 1 \cdot n_e \cdot \mathrm{e} \cdot C_\mathrm{C}(D_\mathrm{P})}{3\eta \cdot Q \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}}$$

Vienna-Type Differential Mobility Analyzer



DMA - Transfer Function


Q_Sh
Q_A
gas stream lines
inner electrode
outer electrode
Q_S
Maximum and minimum
electrical mobility
Q_Ex



Q_{Sh}

Q_A

gas stream lines

inner electrode

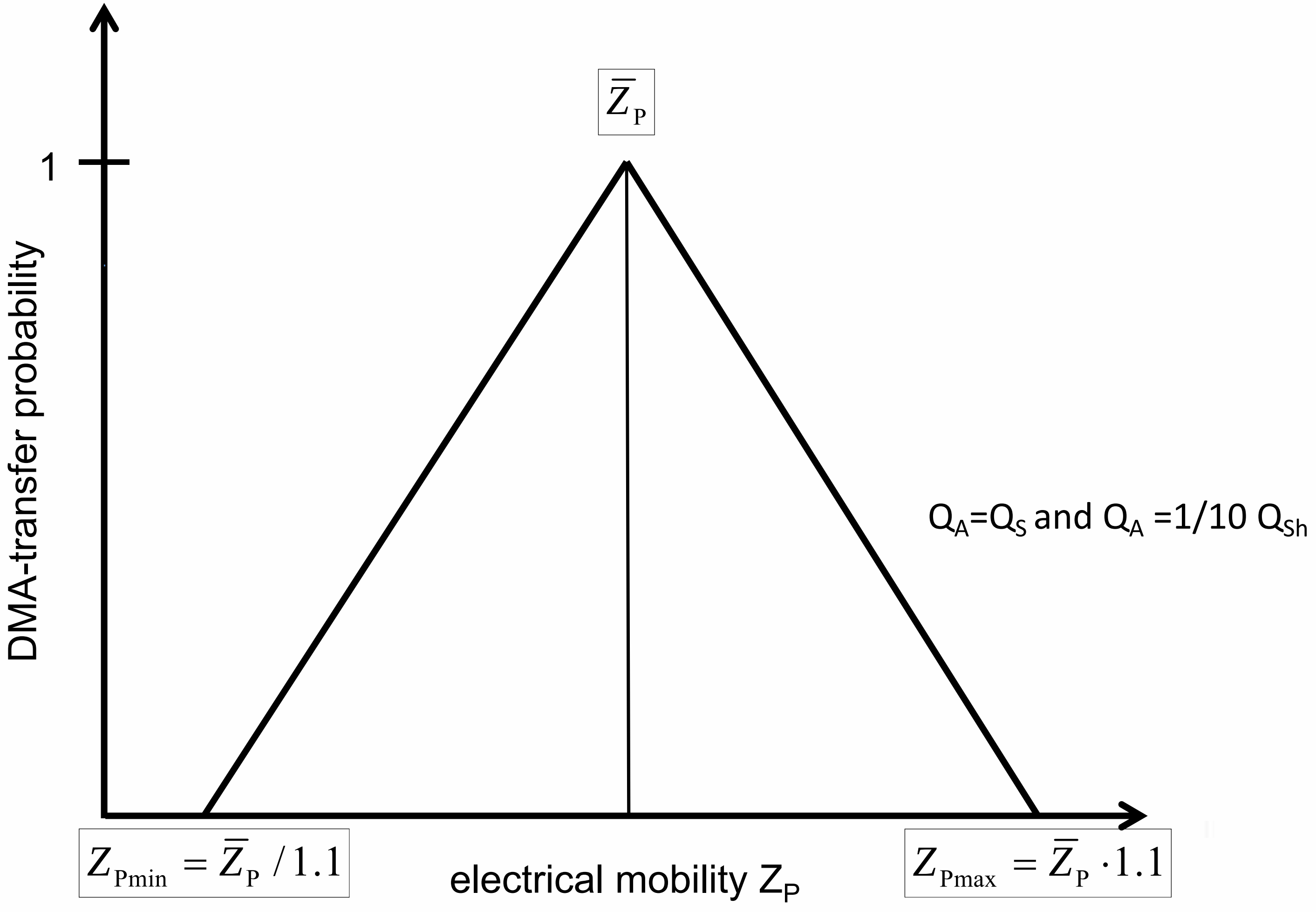
outer electrode

Q_S

mean electrical mobility

Q_{Ex}

DMA - Transfer Function



Q_A=Q_S and Q_A =1/10 Q_{Sh}

- The selected electrical mobility has a certain band width.
- The transfer probability over the mobility bin is however not unity.
- This DMA-transfer function depends on sample flow ratio, and also on the particle size.

Example: $Q_A = Q_s$

- The transfer function has the form of a symmetric triangle.
- The transfer probability of the mean electrical mobility is unity.
- The transfer probability of the upper and lower limit of the mobility bin is Zero.
- For $Q_A > Q_S$ und $Q_A < Q_S$, the transfer function becomes asymmetric. This cases are not discussed, because they are not the standard applications.

- Aerosol particles can be classified due to their electrical mobility in a DMA.
- A volume flow Qs with particles of a defined mobility is taken out of the DMA through a slit at the end of the inner rod.

The mean mobility of these particles can be calculated to:

$$\boxed{\overline{Z}_{\mathrm{P}} = \frac{(Q - \frac{1}{2}(Q_{\mathrm{s}} + Q_{\mathrm{A}})) \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}}$$

The ideal width of the mobility bin is described to:

$$\boxed{\Delta Z_{\mathrm{P}} = \frac{(Q_{\mathrm{s}} + Q_{\mathrm{A}}) \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}}$$

for

$Q_A=Q_S$ and $Q_A = 1/10\ Q_{Sh}$

$$\boxed{\frac{\Delta Z_{\mathrm{P}}}{Z_{\mathrm{P}}} = \frac{1}{5}}$$

$$\boxed{Z_{\mathrm{P}} = \overline{Z}_{\mathrm{P}} \pm 0.1 \cdot \overline{Z}_{\mathrm{P}}}$$

The mean electrical particle mobility can be thus calculated to:

$$\boxed{\bar{Z}_{\mathrm{P}} = \frac{Q_{Sh} \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}}$$

$$\boxed{\frac{n_e \cdot \mathrm{e} \cdot C_{\mathrm{C}}}{3\pi \cdot \eta \cdot \bar{D}_{\mathrm{P}}} = \frac{Q_{Sh} \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2\pi \cdot U \cdot 1}}$$

The voltage to select a the mean electrical particle mobility can be calculated to:

$$\Rightarrow \quad \boxed{U = \frac{3\eta \cdot \bar{D}_{\mathrm{P}} \cdot Q_{Sh} \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}{2 \cdot 1 \cdot n \cdot \mathrm{e} \cdot C_{\mathrm{C}}(\bar{D}_{\mathrm{P}})}}$$

The **mean particle diameter** can be calculated as follows, but it cannot be analytically solved:

$$\Rightarrow \quad \boxed{\bar{D}_{\mathrm{P}} = \frac{2 \cdot U \cdot 1 \cdot n_e \cdot \mathrm{e} \cdot C_{\mathrm{C}}(D_{\mathrm{P}})}{3\eta \cdot Q_{Sh} \cdot \ln(\mathrm{r_o}/\mathrm{r_i})}}$$

Example

D_P= 10 nm

with $Z_P = 2.078 \cdot 10^{-2}$ cm/Vs and $\Delta Z_P = 4.156 \cdot 10^{-3}$ cm/Vs

➔ ΔD_P= 1 nm or D_P =10nm ± 0.5nm

➔ the size resolution is excellent!

The size resolution depends mainly on the ratio of the volume flow rates Q_A/Q_{Sh}.

The greater the ratio, the better becomes the size resolution.

DMA – General Comments

- The particle size range of a mobility particle size spectrometer is defined by geometry of the DMA
- An exact volumetric sheath air flow rate determines a correct sizing.
- The penetration efficiency (transfer function) is size-dependent and has to be considered for particles < 100nm

The reasons are following:

- Diffusion broadening for ultrafine particles
- Particle losses in the aerosol inlet and outlet of the DMA
- The transfer function becomes broader and the maximum transfer probability decreases